Math 625 & Phys 595CL – Homework 6

1) Review the analysis for the bistable climate model performed by John Baez in the following webpage.

http://johncarlosbaez.wordpress.com/2012/11/10/mathematics-and-the-environment-part-6/

Follow the "Temperature Dynamics" link to the numerical simulation which includes an "heat bump" X.

a) τ is the time at which the application of the additional insolation ends. Set it to its maximum value. Vary the coalbedo transition rate γ and the insolation X and report for what combination of values you get two stable climate states with an unstable state in between.

b) Now explore different values of τ and repeat the "experiments" of part a). Do you get two stable equilibria in the long range? If not, ' describe what happens. What happens when $\tau = 0$? Explain your results.

2) In class we went over the latitude dependent model described by

$$R\frac{dT}{dt} = Qs(y)(1-\alpha(y)) - (A+BT) + C(\bar{T}-T)$$

where y is the sine of the latitude. $s(y)$ gives the latitude dependence of the insolation

$$s(y) = 1 - 0.241(3y^2-1) \qquad \int_0^1 s(y)dy = 1$$

$$\bar{T} = \int_0^1 T dy$$

The critical temperature for an ice sheet to form is $T_c = -10°$ C and y_s is the location of the ice line. $T(y_s) = T_c$. The albedo is given by

$$\alpha(y) = \begin{cases} \alpha_2 = 0.62 & y > y_s \\ \alpha_1 = 0.32 & y < y_s \end{cases}$$

$$\alpha(y_s) = (\alpha_1 + \alpha_2)/2$$

Calculate the average albedo

$$\bar{\alpha} = \int_0^1 s(y)\alpha(y)dy$$

for: a) an ice free-Earth, b) an ice-covered Earth, c) an Earth partially covered by ice.

3) Derive the equation for the average temperature $\bar{T}$ by integrating both sides of the equation above over $0 \le y \le 1$ to obtain:

$$R\frac{d\bar{T}}{dt} = Q(1-\bar{\alpha}) - (A+B\bar{T})$$

a) Solve this equation analytically assuming that at $t = 0$ $\bar{T} = \bar{T}_0$.

b) For a numerical analysis consider the values: $Q = 343$ W/m^2; $A = 202$ W/m^2; $B = 1.92$ W/(m^2 °C); $C = 1.6B$; $R = 0.5 \times 10^7$ J/(m^2 °C); $y_s = 0.95$ (72 degrees North).

* Calculate $\tau = B/R$ in units of (1/day) and (1/year).

* From your analytical solution to part a) find the equilibrium solution (as $t \to \infty$). Find the value of the equilibrium temperature for the ice-free and ice-covered cases.

* From your analytical solution plot $\hat{T}(t)$ using τ in years. For the . ice-free case investigate an initial temperature of 30°C and −15°C. For the . ice-covered case investigate an initial temperature of 15°C and −45°C.

4) Please research what is the average Earth's temperature in the equator and in the Noth Pole.

Consider the equation in problem 2) but assume that there is no heat transport term (ie. $C = 0$).

Solve for the equlilibrium temperature. Calculate the equilibrium temperature at the Equator and at the Pole.

By comparing to the average values discuss why is important to include the heat transport term.

5) Here you are asked to repeat steps done in class before performing a calculation.

From problem 2) find the equlibrium temperature T_{eq} by setting $\frac{dT}{dt} = 0$. From problem 3) find the equlibrium average temperature $\bar{T}_{eq}$ by setting $\frac{d\bar{T}}{dt} = 0$.

Substitute $\bar{T}_{eq}$ into the equation for T_{eq} and reorganize the equation to express Q as a function of the other quantities. For $T_{eq} = -10°C$ a possible solution is obtained with $Q = 343$ W/m^2 and $y = y_s = 0.95$ as we have seen.

Keeping $T_{eq} = -10°C$ find Q when the iceline drops to a latitude of 34° North, and the system enters an unstable solution. What percentage is this change from our present day Q?

6) Review the development by North to look for an expression for the equilibrium temperature in terms of a series of spherical harmonics.

$$T(u) = \sum_{n=0} T_n \; P_n(u)$$

$$T_n = \frac{QH_n - A\delta_{n0}}{n(n+1)D + B}$$

$$H_n = \frac{2n+1}{2} \int_{-1}^{1} s(u)a(u)P_n(u) \; dy$$

Assume

$$s(u) = \frac{1}{4}(5 - 3u^2)$$

$$a(u) = a_0 + a_2 \; P_2 \qquad P_2 = (3u^2 - 1)/2$$

For a numerical analysis consider the values: $Q = 342$ W/m^2; $A = 202$ W/m^2; $B = 1.9$ W/(m^2 °C); $D = 0.46B$; $a_0 = 0.679$; $a_2 = -0.241$.

For this model with symmetric hemispheres there are no odd modes in the expansion for the temperature. Calculate T_0 and T_2, and find an expression for $T(u)$ up to that order. Plot your solution as a function of u.