Math625/Phy 595CL – Homework 5

1) For the stress tensor τ, assume that $\tau_{11} = \tau_{22} = \tau_{33} = -p$, where p is pressure. Show that

$$\nabla \cdot \tau = -\nabla p + \vec{F}_{\text{visc}},$$

where part of the problem is to find $\vec{F}_{\text{visc}}$, the viscous (drag) force in terms of components of τ.

2) Let the region $\Omega(t)$ be a material volume at time t for a fluid with velocity field $\vec{u} = \vec{u}(x, y, z, t)$. Define $V(t)$ to be the numerical volume of $\Omega(t)$ (e.g., in units of cubic meters) so that,

$$V(t) = \int_{\Omega(t)} 1 \, dV.$$

a) Show that

$$\frac{d}{dt} V(t) = \int_{\Omega(t)} \nabla \cdot \vec{u} \, dV.$$

b) What is the time evolution of $V(t)$ for an incompressible fluid?

3) Let $\vec{r}$ be the radial vector from the center of the earth to a point on the surface of the earth. Show that the magnitude of centrifugal acceleration, $-\vec{\Omega} \times (\vec{\Omega} \times \vec{r})$, is given by $\Omega^2 R$, where r is the radius of the earth and R is the distance from the point on the surface to the axis of rotation. In which direction does $-\vec{\Omega} \times (\vec{\Omega} \times \vec{r})$ point?

4) Find the Coriolis acceleration experienced by a parcel of air moving with velocity $\mathbf{u} = (u, v, w)$ at the equator.

5) Andrews, problem 4.4

6) Andrews, problem 4.3