Math 625 & Phys 595CL – Homework 4

1) Andrews 3.1

2) Assume that the Earth is a flat thin circular disk of radius 6370 km, orbiting around the Sun at the established distance and with a planetary albedo of 30 %. The vector normal to one face of the disk always points directly to the Sun, and the disk is made of perfectly conducting material, so both faces of the disk are at the same temperature.

Review the principles for the calculation that led to equation 1.24. Repeat the calculation for the case of the disk Earth and find its temperature.

3) Consider the case in which only absorption contributes to the extinction coefficient k_ν.

In a highly rarefied gas a molecule in a given quantum state radiates with a frequency ν_0. If the molecule has a line of sight speed $u << c$, the Doppler effect gives a shift to the observed frequency:

$$\nu = \nu_0(1 \pm u/c)$$

From the kinetic theory of gases if the system is in thermodynamic equilibrium the probability that the speed lies between u and $u + du$ is

$$P(u) = \left(\frac{m}{2\pi k_B T}\right)^{1/2} \exp\left(-\frac{mu^2}{2k_B T}\right)$$

where m is the mass of the molecules and T the temperature.

The line-shape function $f(\nu - \nu_0)$ is defined as:

$$f(\nu - \nu_0) = A \int_{-\infty}^{\infty} P(u)\ \delta(\nu - \nu_0(1 + u/c)\ )\ du$$

where A is a normalization constant.

The extinction coefficient is given by

$$k_\nu = S f(\nu - \nu_0)$$

where S is the line strength.

With the normalization

$$\int_{-\infty}^{\infty} f(\nu - \nu_0) d\nu = 1$$

show that

$$k_\nu = \frac{S}{\gamma_D \sqrt{\pi}} \exp\left(-\frac{(\nu - \nu_0)^2}{\gamma_D^2}\right)$$

and find γ_D, the half-width at half maximum.

4) Return to Lambert's law, represented by equation 3.8. Notice that there are no sources present. Assume that the beam incident at the top of the atmosphere has spectral irradiance $L_{\nu\infty}$ which makes an angle $\theta = 127°$ with the normal direction to the surface of the Earth.

Assume that the spectral irradiance k_ν and the density of active absorbers ρ_a only depend on z the vertical distance.

a) Change variables and express equation 3.8 as a function of z. Integrate the equation from the top of the atmosphere ($z = \infty$) to a level (z). Identify the scaled optical depth and the transmissivity of the layer $T_\nu(z)$ in the solution

$$L_\nu(z) = L_{\nu\infty}\, T_\nu(z, \infty)$$

b) Generalize the transmissivity for a beam passing through a layer between heights z and z' and identify an expression for the optical thickness between these levels. Assume $\rho_a = 0.1$ kg/m^3, and a layer of thickness 100 m. For constant extinction coefficient of values $k_\nu = 10^{-3}$ m^2/kg and 10^{-1} m^2/kg, calculate the optical thickness, and the absorptivity $\alpha_\nu = 1 - T_\nu$.