Math 625/Phys 595CL – Homework 3

1) Andrews 2.10

2) Part c) of this exercise will be used in lecture to derive the formula for $B(T)$, the blackbody radiance at temperature T, and for the derivation of the Stephan-Boltzmann law for blackbody radiation.

a) Find the Fourier series for $f(x) = x^2$ on $[-\pi, \pi]$,

$$x^2 = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx),$$

i.e., find all a_n and b_n.

b) Use part a) and Parseval's Identity, which says,

$$\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)^2 dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty}(a_n^2 + b_n^2),$$

to find the exact sum of the series,

$$\zeta(4) = \sum_{n=1}^{\infty}\frac{1}{n^4}.$$

(This is the Riemann-zeta function evaluated at 4.)

c) Prove that

$$\int_0^{\infty} \frac{x^3}{e^x - 1} dx = \frac{\pi^4}{15}$$

by first showing that

$$\frac{1}{e^x - 1} = \sum_{n=1}^{\infty} e^{-nx}$$

for $x > 0$, and then using integration by parts along with part b).

3) The purpose of this exercise is increase familiarity with the Transport Theorem (or Liebniz Rule) used for radiative transfer and later for atmospheric dynamics.

a) Derive the one-dimentional version, i.e., if $f(x,t), g(t), h(t)$ are continuously differentiable show that,

$$\frac{d}{dt}\int_{g(t)}^{h(t)} f(x,t)dx$$

is given by the right hand side of Eq. 1.1 of Flander's article linked from the course webpage.

b) Use the Divergence Theorem to show that the two versions of the Transport Theorem given in lecture are the same.