Math 625/Physics 595CL – Homework 2

1) When the internal energy, U, is given as a function of volume and temperature, i.e., $U = U(V, T)$, show that:

$$\delta Q = \left(\frac{\partial U}{\partial T}\right)_V dT + \left[\left(\frac{\partial U}{\partial V}\right)_T + P\right] dV$$

and

$$\left(\frac{\Delta Q}{\Delta T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V$$

as discussed in class in connection with the definition of heat capacity at constant volume.

2) Andrews 2.1. You need only find the total enthalpy of the atmosphere as a function of temperature, which you may assume to be constant. Approximate the density of sea water by the density of pure water and assume that the surface area of the oceans is 70.9% of the surface area of Earth, or 3.61×10^8 km^2. Use data from the appendix (pg 225-6) as needed.

3) Andrews 2.3

4) Assume (as usual) that the gravitational acceleration g is constant. Prove that if the potential temperature is constant throughout an atmosphere consisting of an ideal gas in hydrostatic balance, then temperature must be a linear function of height with DALR lapse rate. Now using the result of Andrews 2.3, do Andrews 2.4.

5) During an evening, long wave radiative heat emissions cause a dry air parcel to descend so that its pressure increases from 900 hPa to 910 hPa, and its entropy decreases by 15 J kg^{-1} K^{-1}. If its initial temperature is 280 K, determine the parcel's a) final temperature and b) final potential temperature.