Physics 595CL – Homework 4

1) A small blackbody satellite orbits the Earth at sufficient height so that the flux density of Earth's radiation is negligible compared to the solar radiation. Suppose that the satellite suddenly passes into the Earth's shadow. At what rate will it initially cool? The satellite has a mass $m = 103$ kg and a specific heat $c = 10^3$ J/(kg K), and it is spherical with a radius $r = 1$ meter, and the temperature is uniform over its surface.

2) In the plane parallel model of the atmosphere, let $F_\nu(z) = F_\nu^{\uparrow}(z) - F_\nu^{\downarrow}(z)$ denote the total spectral irradiance, where $F_\nu^{\uparrow}(z)$ and $F_\nu^{\downarrow}(z)$ are given by Eq. (3.33) and the equation that immediately follows on page 78 of your textbook. Use Liebnitz' rule to show that the radiative diabatic heating rate, $Q_\nu(z)$, is given by,

$$
\begin{aligned}
-\frac{\rho(z)Q_\nu(z)}{\pi} &= B_\nu(z)\frac{d\mathcal{T}_\nu^*(z,\infty)}{dz} + [B_\nu(0) - B_\nu(z)]\frac{d\mathcal{T}_\nu^*(z,0)}{dz} \\
&+ \int_0^z [B_\nu(z') - B_\nu(z)]\frac{\partial^2 \mathcal{T}_\nu^*(z,z')}{\partial z\,\partial z'}dz' \\
&+ \int_z^\infty [B_\nu(z') - B_\nu(z)]\frac{\partial^2 \mathcal{T}_\nu^*(z,z')}{\partial z\,\partial z'}dz'
\end{aligned}
$$

3) Consider the case in which only absorption contributes to the extinction coefficicient k_ν.

In a highly rarefied gas a molecule in a given quantum state radiates with a frequency ν_0. If the molecule has a line of sight speed $u << c$, the Doppler effect gives a shift to the observed frequency:

$$\nu = \nu_0(1 \pm u/c)$$

From the kinetic theory of gases if the system is in thermodynamic equilibrium the probability that the speed lies between u and $u + du$ is

$$P(u) = \left(\frac{m}{2\pi k_B T}\right)^{1/2} \exp\left(-\frac{mu^2}{2k_B T}\right)$$

where m is the mass of the molecules and T the temperature.

The line-shape function $f(\nu - \nu_0)$ is defined as:

$$f(\nu - \nu_0) = A \int_{-\infty}^{\infty} P(u)\ \delta(\nu - \nu_0(1 + u/c)\ )\ du$$

where A is a normalization constant.

The extiction coefficient is given by

$$k_\nu = Sf(\nu - \nu_0)$$

where S is the line strength.

With the normalization

$$\int_{-\infty}^{\infty} f(\nu - \nu_0) d\nu = 1$$

show that

$$k_\nu = \frac{S}{\gamma_D \sqrt{\pi}} \exp\left(-\frac{(\nu - \nu_0)^2}{\gamma_D^2}\right)$$

and find γ_D, the half-width at half maximum.

4) Read carefully Section 3.7.2 and Section 3.8 (fill in on your own, but do not turn in, all the steps for the equations). Do Andrews' problem 3.11