Math 396 CL Homework #1

1) W+H 3.19

$M_C = 12 \frac{g}{mol}$
$M_O = 16 \frac{g}{mol}$
$M_N = 14 \frac{g}{mol}$

95% CO_2 and 5% N_2

VA = Venusian atmosphere

$$M_{VA} = 0.95\left(12 + 2(16)\right) + 0.05\left(2(14)\right) \ \frac{g}{mol}$$

$$M_{VA} = 43.2 \ \frac{g}{mol}$$

$$R_{VA} = \frac{R^*}{M_{VA}} = \frac{\left(8.3145 \ \frac{J}{K\,mol}\right)}{43.2 \ \frac{g}{mol}} \times \left(\frac{1000g}{Kg}\right) = 192.5 \ \frac{J}{K\,Kg}$$

2) W+H 3.20

Express in terms of T_v

$p = R_d \rho T_v$ $\qquad R_d = 287 \ \frac{J}{Kg\,K}$

Alternatively from the direct application of the ideal gas law

$$p = R_{moist} \rho T$$

$$M_{moist} = (0.99) M_{dry} + (0.01) M_{wv}$$

$$M_{moist} = \left[(0.99)(28.97) + (0.01)(18)\right] \frac{g}{mol} = 28.86 \ \frac{g}{mol}$$

$$R_{moist} = \frac{R^*}{M_{moist}} = \frac{8.3145 \ \frac{J}{K\,mol}}{28.86 \ \frac{g}{mol}} \times \frac{1}{\left(\frac{1\,Kg}{1000\,g}\right)} = 288.1 \ \frac{J}{Kg\,K}$$

$$R_d \rho T_v = R_{moist} \rho T$$

$$T_v = \frac{R_{moist}}{R_d} T = \left(\frac{288.1}{287}\right) T = 1.0038\ T$$

or substituting directly into eq. 3.16

$$T_v = \frac{T}{\left(1 - \frac{e}{p}(1-\varepsilon)\right)}$$

$e = 0.01\,p$

$$T_v = \frac{T_v}{\left(1 - 0.01(1-0.622)\right)} = 1.0038\,T = (1 + 0.0038)\,T$$

$$\rightarrow \quad T_v - T = 0.0038\,T$$

3) W + H 3.26 For $T = 288\,K$ $T_v - T$

$\longleftarrow$ $p_2 = 200\,hPa$ $\longrightarrow$ $z_2 = ?$ p_2

$z_1 = 0\,m$ (sea level)

$\longleftarrow$ $p_1 = 940\,hPa$ $\longrightarrow$ $\longleftarrow$ $p_1 = 1010\,hPa$ $\longrightarrow$

hurricane outside

$T_{outside} = -3\,°C = 270\,K$

- First calculate z_2 from the information outside

$$z_2 - z_1 = \frac{R_d \overline{T}}{g} \ln\left(\frac{p_1}{p_2}\right)$$

$$z_2 = \frac{(287)(270)}{9.8} \ln\left(\frac{1010}{200}\right)\,m = 12{,}790\,m$$

- Now use this height to calculate the $\overline{T}$ inside the hurricane

$$\overline{T}_{in} = \frac{g}{R_d}\, z_2\, \frac{1}{\ln\left(\frac{p_1}{p_2}\right)}$$

$$\bar{T}_{in} = \frac{(9.81)}{(270)} (12{,}790) \frac{1}{\ln\left(\frac{940}{200}\right)} \text{ K} = 282.5 \text{ K}$$

$$\Delta T = \bar{T}_{in} - \bar{T}_{out} = 282.5 - 270 = 12.5 \text{ K}$$

4) W+H 3.32

$T = 15°C \qquad T = (273.15 + 15)\text{ K} = 288.15\text{ K}$

$m = 2\text{kg}$

$V_2 = \frac{1}{10} V_1 \qquad V_1 = 10 V_2$

$$pV = m R_d T$$

• Work done on a system

$$dW = -p\,dV = -\frac{m R_d T}{V} dV$$

$$W = -m R_d T \int_{V_1}^{V_2} \frac{dV}{V} = -m R T \ln\left(\frac{V_2}{V_1}\right)$$

$$W = -(2\text{ kg}) \left(287 \frac{\text{J}}{\text{K}\cdot\text{kg}}\right) (288.15\text{ K}) \ln\left(\frac{V_2}{10 V_2}\right) =$$

$$W = 3.8 \times 10^5 \text{ J}$$

5) W+H 3.33

$p V^{\gamma} = \text{constant} \qquad \gamma = \frac{c_p}{c_v}$

$pv = RT \qquad (3.3)$

$dq = c_v\,dT + p\,d\alpha \qquad (3.41)$

$c_p = c_v + R \qquad (3.45)$

a) For an adiabatic transformation $dq=0$

$$d(p\alpha) = R\,dT$$

$$dp\,\alpha + p\,d\alpha = R\,dT$$

$$dT = -\frac{p\,d\alpha}{c_v}$$

$$dp\,\alpha + p\,d\alpha = -\frac{R}{c_v}\,p\,d\alpha$$

$$dp\,\alpha + p\,d\alpha\left(1+\frac{R}{c_v}\right) = 0$$

$$c_v\,dp\,\alpha + p\,d\alpha\,(c_v+R) = 0$$

$$c_v\,dp\,\alpha + c_p\,p\,d\alpha = 0$$

$$\alpha = \frac{V}{m}$$

$$c_v\,dp\,V + c_p\,p\,dV = 0$$

$$dp\,V + \frac{c_p}{c_v}\,p\,dV = 0$$

$$\frac{dp}{p} = -\left(\frac{c_p}{c_v}\right)\frac{dV}{V} = -\gamma\frac{dV}{V}$$

$$\ln p = -\gamma \ln V + c \qquad c = \text{constant}$$

$$p = V^{-\gamma}c' \qquad c' = \text{constant}$$

$$pV^{\gamma} = c'$$

6) W+H 3.36

① $\theta = T\left(\frac{p_0}{p}\right)^{K}$ $\quad K = \frac{R}{c_p}$

② $pV^{\gamma} = \text{constant}$ $\quad \gamma = \frac{c_p}{c_v}$

③ $pV = R^{*}T$

• From ② & ③ $p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$ ④

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \quad ⑤$$

$$\frac{V_1}{V_2} = \frac{p_2}{T_2}\frac{T_1}{p_1} \quad ⑥$$

• Using ⑥

$$\frac{p_1}{p_2} = \left(\frac{V_2}{V_1}\right)^{\gamma} = \left(\frac{T_2 \, p_1}{p_2 \, T_1}\right)^{\gamma} \quad ⑦$$

Then

$$\left(\frac{p_1}{p_2}\right)^{1-\gamma} = \left(\frac{T_2}{T_1}\right)^{\gamma} \quad ⑧$$

And

$$\frac{T_2}{T_1} = \left(\frac{p_1}{p_2}\right)^{\frac{1-\gamma}{\gamma}}$$

$$\frac{1-\gamma}{\gamma} = \frac{1 - \frac{c_p}{c_v}}{\frac{c_p}{c_v}} = \frac{c_v - c_p}{c_p} = \frac{c_v - (R + c_v)}{c_p} = -\frac{R}{c_p} = -K$$

$$\frac{T_2}{T_1} = \left(\frac{p_1}{p_2}\right)^{-K} = \left(\frac{p_2}{p_1}\right)^{K}$$

• From ①

$$\theta_1 = T_1\left(\frac{p_0}{p_1}\right)^{K} \qquad \theta_2 = T_2\left(\frac{p_0}{p_2}\right)^{K}$$

$$\longrightarrow \quad \frac{\theta_2}{\theta_1} = \frac{T_2\left(\frac{p_0}{p_2}\right)^{K}}{T_1\left(\frac{p_0}{p_1}\right)^{K}} = \frac{T_2}{T_1}\left(\frac{p_1}{p_2}\right)^{K} = 1 \quad \longrightarrow \quad \theta_2 = \theta_1$$

$\theta = \text{constant}$