Extras, Assignment 2
Math 651b & Phys 640

Problem I. The 3-sphere, $S^3(R)$, of radius R is the hypersurface in $\mathbb{R}^4$ given in Cartesean coordinates x, y, z, w by,

$$x^2 + y^2 + z^2 + w^2 = R^2.$$

Polar coordinates in $\mathbb{R}^4$ are given by,

$$\begin{aligned} w &= R\cos\chi \\ z &= R\sin\chi\cos\theta \\ y &= R\sin\chi\sin\theta\sin\phi \\ x &= R\sin\chi\sin\theta\cos\phi, \end{aligned}$$

where χ and θ range from 0 to π and ϕ ranges from 0 to 2π. Derive an expression for the metric on $S^3(R)$ induced by the usual dot-product (i.e., the Euclidean metric) on $\mathbb{R}^4$ and express it in terms of χ, θ, ϕ. (For the answer, see prob. 2.9, page 52 of the text).

Problem II. Find the volume of $S^3(R)$.

Problem III. Find the area of the 2-sphere defined by $\chi = \chi_0$.