



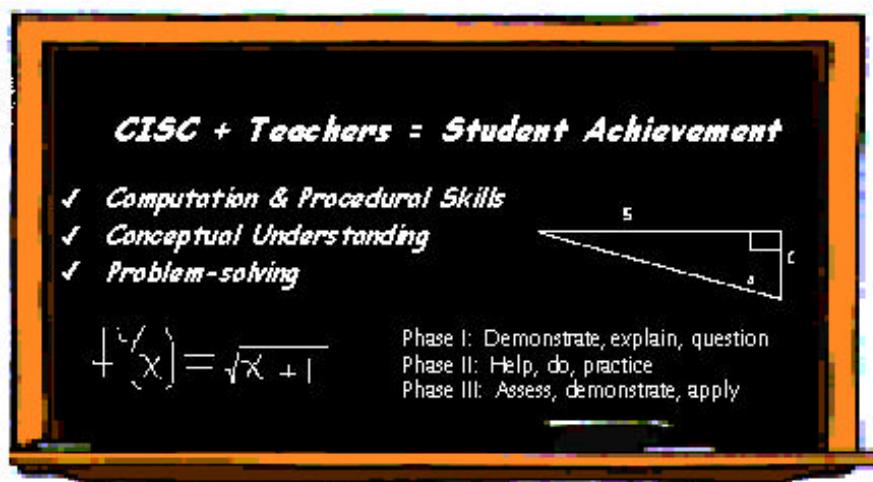
CISC - Curriculum & Instruction Steering Committee

California County Superintendents Educational Services Association

Primary Content Module II NUMBER SENSE: Whole Numbers and Integers

The Winning EQUATION

A HIGH QUALITY MATHEMATICS PROFESSIONAL DEVELOPMENT
PROGRAM FOR TEACHERS IN GRADES 4 THROUGH ALGEBRA II



STRAND: NUMBER SENSE: Whole Numbers and Integers

MODULE TITLE: PRIMARY CONTENT MODULE II

MODULE INTENTION: The intention of this module is to inform and instruct participants in the underlying mathematical content in the areas of whole numbers and integers.

THIS ENTIRE MODULE MUST BE COVERED IN-DEPTH.

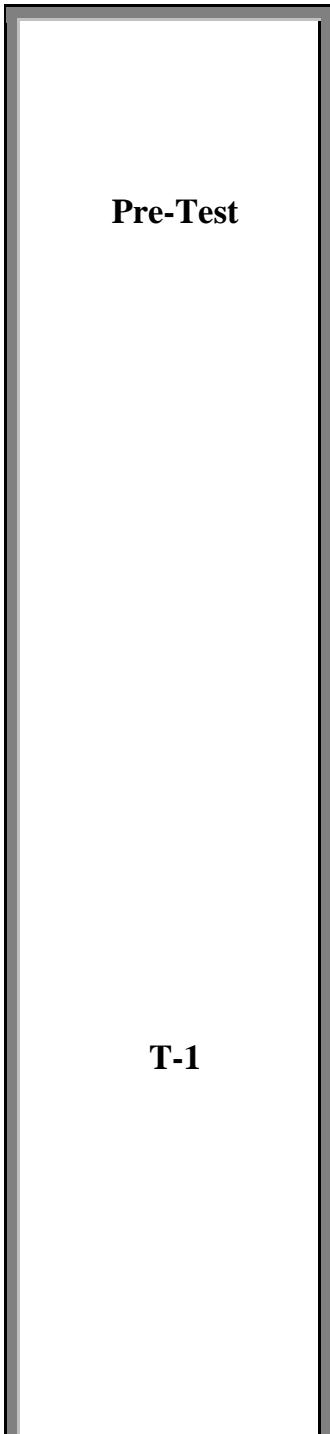
The presentation of these Primary Content Modules is a departure from past professional development models. The content here, is presented for individual teacher's depth of content in mathematics. Presentation to students would, in most cases, not address the general case or proof, but focus on presentation with numerical examples.

In addition to the underlying mathematical content provided by this module, the facilitator should use the classroom connections provided within this binder and referenced in the facilitator's notes.

TIME: 2 hours

PARTICIPANT OUTCOMES:

- Demonstrate understanding of the base 10 number system.
- Demonstrate understanding of place value and powers of ten.
- 3. • Demonstrate how to use the commutative and associative laws for addition and their use with subtraction of natural numbers, whole numbers, and integers.
- Demonstrate how to use the basic algorithms for addition and subtraction of natural numbers, whole numbers, and integers.



Pre-Test

T-1

PRIMARY CONTENT MODULE II

NUMBER SENSE: Whole Numbers and Integers

Facilitator's Notes

Ask participants to take the Pre-Test. After reviewing the results proceed with the following two lessons with the first lesson on whole numbers and integers and the second lesson on multiplication and division of whole numbers.

This section focuses on our Hindu-Arabic number system and the operations of addition and subtraction. It also reviews the standard algorithms for + and – and shows that the commutative, associative, and distributive¹ laws along with our base-10 positional number system are what makes them work.

Even though the concepts "number", "addition" and "subtraction" arise in grades K-3, it is important for teachers at grades 4-7 to be aware of the structures that they entail. Accordingly, it seems desirable to spend an hour "reminding" participants of the concepts that underlie counting numbers, addition, subtraction, and integers.

Notation: Arithmetic begins with a counting litany

one, two, three, ... or uno, dos, tres, ... or ...

whose words correspond to **counting numbers** (or **natural numbers**). Children arrive at school with an innate understanding of the first few of these (unity, pair, triad) which they soon learn to represent in terms of the symbols 1, 2, 3,

The question

"What is a counting number?"

can be a challenging one to answer precisely. You may even wonder why such a question should be asked, since virtually everyone is able to count.

A reason for addressing such fundamental questions is to encourage teachers to develop a methodical understanding of numbers, one that will make us more effective in explaining the number concept to children. In this context, and for the purposes of teaching arithmetic, a good answer is,

"A counting number is an attribute of a nonempty set."

¹ Sometimes referred to as CAD.

**T-2**

Tell participants that a set is a collection of objects. Examples: {1, 2, 3, 4, 5}; {red, yellow, blue}; {-2, -1, 0, 1, 2}. That is, just as length is an attribute of a line segment and area is an attribute of a plane shape, so does the set (collection) of people in a room have an attribute that is readily described in terms of the symbols 1, 2, 3,

While natural numbers may be difficult to define, they can be **characterized** in terms of their properties. Whether we are counting in English or Spanish or ... :

1. There is a first counting number, e.g., "one," "uno."
2. Every counting number has a successor.
3. No two counting numbers have the same successor.
4. "One" is not the successor of any counting number.
5. The successors of "one" exhaust the counting numbers.

These five properties paraphrase "axioms for the natural number system" given by Giuseppe Peano in the late 19th century. From these five assumptions alone, it is possible to deduce many of the important properties of numbers, such as the commutative, associative, and distributive properties, explained in what follows.

T-3

One of the advantages of Hindu-Arabic notation is that it enables us to verbalize large numbers. That is, "three hundred fifty four" is just a verbalization of

$$\begin{aligned} 354 &= 300 + 50 + 4 \\ \text{or} \quad 354 &= 3 \times 10^2 + 5 \times 10^1 + 4 \times 10^0 \end{aligned}$$

The base 10 system makes it easy to compare two counting numbers, e.g., to decide that

$$1047 > 698 \text{ and } 6297 < 6302.$$

Introduce the following transparencies to discuss place value, powers of ten and large and small numbers.

T-4

Uncover T-4 in stages to the bottom of each arrow. This is another illustration of the base ten system.

T-5 and H-5

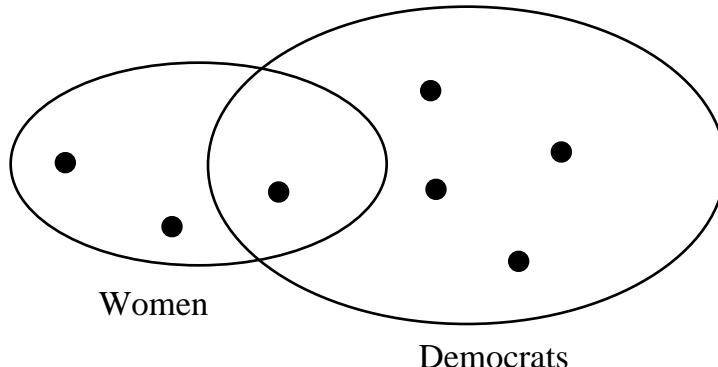
Have participants use H-5 to explain the meaning of the symbol represented by 2,463. Use T-5 to record some suggestions from participants.

T-6

Use T-6 to discuss the powers of 10. Place value is also displayed in columns, representing the "Secret code for our number system."

**T-7**

Addition: If counting numbers are conceived as attributes of a set, then addition corresponds to "combining two sets" (this is also referred to as "forming the union of two sets"). Essential to this interpretation of " $3 + 5 = 8$ " is that there be no overlap in the sets (i.e., that the sets be "disjoint"). If there are 3 women and 5 democrats in a room, then combining the set of women with the set of democrats need **not** correspond to simple addition:



Only if there is no overlap between these sets does "combining sets" of size 3 and 5 correspond to " $3 + 5 = 8$." This is the basis for a **definition** of addition.

With this concept established, we turn to two properties of addition:

Commutative Law of Addition: for any numbers a and b ,

$$a + b = b + a$$

Associative Law² of Addition: for any numbers a , b , and c ,

$$(a + b) + c = a + (b + c).$$

While interpreting addition in terms of "combining sets without overlap" makes these properties eminently reasonable, we will make no effort to "prove" them. Instead, they will be taken as **assumptions** (axioms) that underlie arithmetic. One reason they are important is that they enable us to take advantage of the Hindu-Arabic number system to develop **algorithms** with which we can add and subtract large numbers with ease.

T-8

Let us consider the standard algorithm for solving the addition problem " $354 + 167$ " in the form

$$\begin{array}{r} 354 \\ + 167 \\ \hline 521 \end{array}$$

² Based on the Associative Law, we often write sums of three or more numbers without including any parentheses.

**T-10**

What is going on when you sum two numbers in this way?

H-10

Hand out H-10
The answer lies in reconciling this process with one that implements the commutative and associative laws as they apply to $354 + 167$ expressed in base-10 positional notation. That is, writing the problem as

$$(300 + 50 + 4) + (100 + 60 + 7)$$

these laws enable us to regroup and to obtain

$$\begin{aligned}(300 + 100) + (50 + 60) + (4 + 7) &= 400 + 110 + 11 \\ &= 400 + (100 + 10) + (10 + 1) \\ &= (400 + 100) + (10 + 10) + 1\end{aligned}$$

H-10A

Pass out H-10A and give time to work on it.

H-10B

Answers to H-10B.

T-11

How are commutative and associative laws used in the addition algorithm?

$$\begin{array}{r} 52 \\ +34 \\ \hline 86 \end{array}$$

$52 + 34 = (50 + 2) + (30 + 4)$ 52
 = $[(50 + 2) + 30] + 4$ +34
 = $[50 + (2 + 30)] + 4$ Associative
 = $[50 + (30 + 2)] + 4$ Associative
 = $[(50 + 30) + 2] + 4$ Commutative
 = $(50 + 30) + (2 + 4)$ Associative
 = $80 + 6$ Associative
 = 86

Important: Addition facts for sums to 20 are to be memorized in first grade.

California Mathematics Standard: Number Sense 2.1, Grade 1.

It is this series of operations that the addition algorithm enables children to perform in a remarkably compact and simple format.

H-11

Pass out H-11. Go over step by step solutions.

H-11A

Answers to H-11.

**T-12**

Subtraction: In the same way as addition corresponds to the intuitive operation of "putting together," so does subtraction arise as the intuitive inverse operation of "taking away." In this context, one could argue that the reason that

$$12 - 7 = 5 \text{ is because } 12 = 7 + 5.$$

Indeed, this can be used as the **definition** of subtraction. That is

$$a - b = c \text{ if and only if } a = b + c.$$

This definition codifies the correspondence between subtraction and "taking away" and leads to the standard algorithm for implementing subtractions in the case of large numbers.

T-13

For example, to compute $728 - 253$, we write

$$\begin{array}{r} 728 \\ - 253 \\ \hline 475 \end{array}$$

In order to arrive at the correct answer we,

1. Subtract the digits in the units column, $8 - 3 = 5$. Record the 5 in the units place.
2. Subtract the 5 from the 2 in the tens column by regrouping one of the 7 hundreds to make it 12 tens; then subtract $12 - 5 = 7$. Record the 7 in the tens place.
3. Subtract the 2 from the 6 that is left over in the hundreds column; $6 - 2 = 4$.

What is going on when you arrive at the correct answer in this way? Anticipating some of the discussion to follow (i.e., that it is possible to extend the commutative, associative, and distributive laws from the counting numbers to the integers), we would again write the problem as

$$\begin{aligned} (700 + 20 + 8) - (200 + 50 + 3) &= (700 - 200) + (20 - 50) + (8 - 3) \\ &= (600 - 200) + (120 - 50) + (8 - 3) \\ &= 400 + 70 + 5 = 475 \end{aligned}$$

It is this series of operations that the subtraction algorithm enables children to perform in a remarkably compact and simple format.

**T-14**

Zero and Additive Inverses: Zero is the additive identity.

$$\begin{aligned}0 + a &= a \\a + 0 &= a\end{aligned}$$

for any number a .

Whole Numbers:

The set of whole numbers (W) is the set of counting numbers (i.e., natural numbers) together with zero.

$$W = \{0, 1, 2, 3, \dots\}$$

Additive Inverse and Negative Numbers

Definition: If a is a whole number, $-a$ is defined to be the number satisfying:

$$-a + a = 0$$

and

$$a + -a = 0$$

Example: If $a = 2$, $-a = -2$,

$$2 + -2 = 0$$

$-0 = 0$ because $0 + 0 = 0$.

T-15

With the definition of additive inverse for every whole number, a new set of numbers is formed.

The integers

$$Z = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

The integers may be described as the set of whole numbers and their additive inverses.

Note that $N \subset W \subset Z$.

T-16 and T-17

Review slide with participants. The arrows may be displayed on or above the number line.

T-18 and H-18

Practice.

T-19

Define additive inverse for integers.

**T-20**

Answers:

- 1) $2 + -6 = -4$, $2 - 6 = -4$
- 2) $-3 + 5 = 2$, $5 - 3 = 2$
- 3) $-5 + 7 = 2$, $7 - 5 = 2$
- 4) $-6 + 4 = -2$, $4 - 6 = -2$

**T-21 and H-21
T-21A and H-21A****T-22
T-23****Post-Test**

This is an exploration to verify if the Commutative and Associative Laws hold for the set of integers. **Answer: yes**

Optional Recreational Problem

Ask participants to take the post-test.

Pre-Test

1. What is a natural number? _____

2. What is a whole number? _____

3. What is an integer? _____

4. Does $(19 - 5) - 3 = 19 - (5 - 3)$? Explain your answer.

5. Write 6,742,983 in base 10 exponential notation.

6. What is the value of each “3” in 930,346,253?

In problems 7 – 9, use the examples of students work and explain the errors and the misunderstanding of fundamental conceptual principles.

7.
$$\begin{array}{r} 4527 \\ +319 \\ \hline 7717 \end{array}$$

8.
$$\begin{array}{r} 3812 \\ -764 \\ \hline 3152 \end{array}$$

9. $6351 < 999$

Pre-Test Answer Key

1. Any number in the set $\{1, 2, 3, 4, \dots\}$
2. Any number in the set $\{0, 1, 2, 3, \dots\}$
3. Any number in the set $\{\dots -3, -2, -1, 0, 1, 2, 3, \dots\}$
4. No! $(19 - 5) = 14 - 3 = 11$ $(5 - 3) = 2$ and $19 - 2 = 17$.
5. $6 \cdot 10^6 + 7 \cdot 10^5 + 4 \cdot 10^4 + 2 \cdot 10^3 + 9 \cdot 10^2 + 8 \cdot 10^1 + 3 \cdot 10^0$
6. From left to right: 30,000,000 thirty million
 300,000 three hundred thousand
 3 three
7. 319 is not lined up according to the correct place value. The 9 must be under the 7 (one's place); the 1 must be under the 2 (ten's place); the 3 must be under the 5 (hundred's place).
$$\begin{array}{r} 4527 \\ +319 \\ \hline 4846 \end{array}$$
8. The error in this problem is that the subtraction was done without regard to place value and in each column the smaller digit was subtracted from the larger digit.
$$\begin{array}{r} 3812 \\ -764 \\ \hline 3048 \end{array}$$
9. The error in this problem is that the student used the size of the digit "9" instead of considering the place value.
$$6351 > 999$$

The question:

“What is a counting number?”

“A counting number is an attribute of a nonempty set.”

- just as length is an attribute of a line segment
- area is an attribute of a plane shape
- so does the set (collection) of people in a room have an attribute that is readily described in terms of the symbols 1, 2, 3, ...

Natural numbers can be **characterized** in terms of their properties.

1. There is a first counting number, e.g., "one" or "uno."
2. Every counting number has a successor.
3. No two counting numbers have the same successor.
4. "One" is not the successor of any counting number.
5. The successors of "one" exhaust the counting numbers.

From these five assumptions alone, it is possible to deduce many of the important properties of numbers; such as the commutative, associative, and distributive properties, explained in what follows.

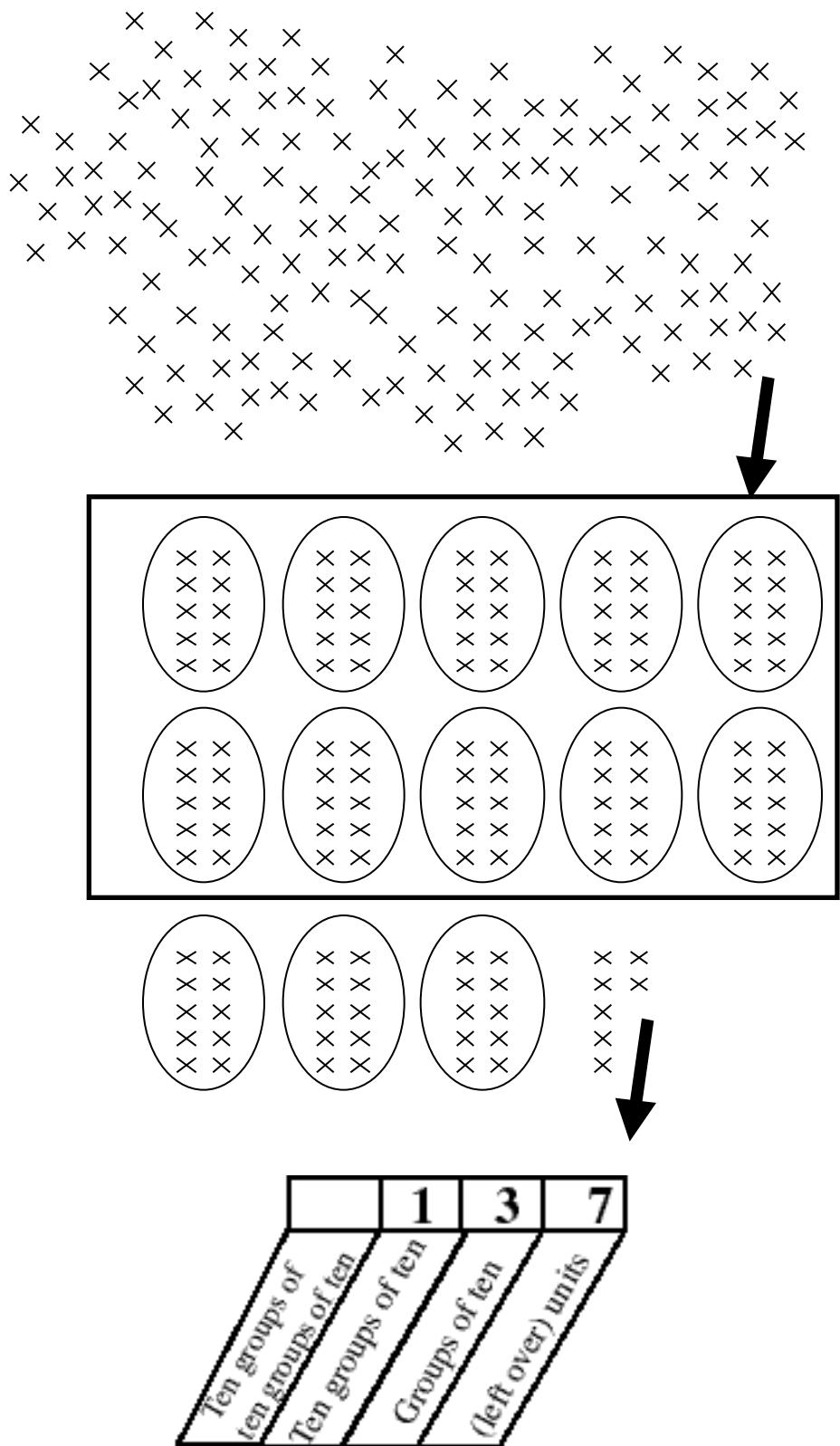
"three hundred fifty four" is a verbalization of

$$354 = 300 + 50 + 4$$

$$354 = 3 \times 10^2 + 5 \times 10^1 + 4 \times 10^0$$

The base 10 system makes it easy to compare two counting numbers, e.g., to decide that

$$1047 > 698 \text{ and } 6297 < 6302$$



Whole Numbers and Integers

Place Value

Worksheet

What is the meaning of this symbol?

2,463

What is the meaning of this symbol?

2,463

Answer:

$$2,463 = 2 \times 10^3 + 4 \times 10^2 + 6 \times 10^1 + 3 \times 10^0$$

or

two groups of 1000, 4 groups of 100,
6 groups of 10, and 3 units.

Powers of 10

$$10^0 = 1$$

$$10^1 = 10$$

$$10^2 = 100$$

$$10^3 = 1,000$$

$$10^4 = 10,000$$

Place Value Columns

“Secret Code for Our Number System”

ten thousands	thousands	hundreds	tens	ones
10^4	10^3	10^2	10^1	10^0
3	5	6	4	7

$$35,647 = 3 \times 10^4 + 5 \times 10^3 + 6 \times 10^2 + 4 \times 10^1 + 7 \times 10^0$$

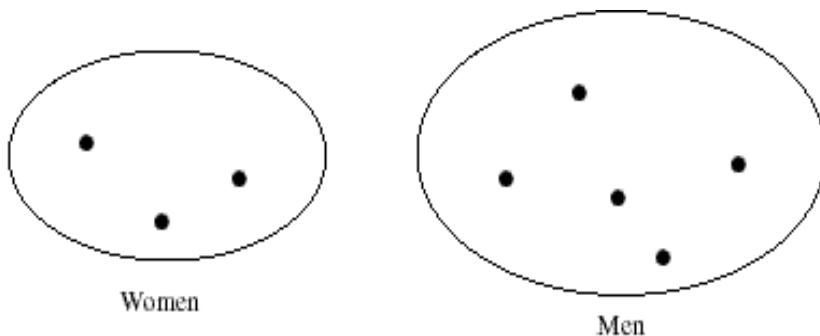
$$35,647 = 30000 + 5000 + 600 + 40 + 7$$

Addition

Addition of counting numbers corresponds to combining two sets (or taking the “union” of two sets.) It is essential that the sets are disjoint. They must not overlap.

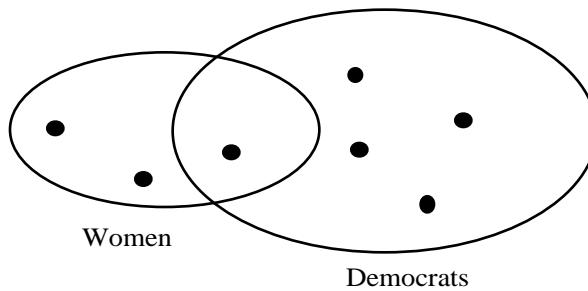
Example: $3 + 5 = 8$

3 women and 5 men are 8 people



Question: If there are 3 women and 5 Democrats in a room, are there necessarily 8 people present?

Maybe not.



This example does not model addition because of the overlap.

Properties of Numbers

Addition

Commutative Law of Addition: For any numbers a and b ,

$$a + b = b + a$$

Associative Law of Addition: For any numbers a , b , and c ,

$$(a + b) + c = a + (b + c)$$

Standard Algorithm for Addition

To solve $354 + 167 =$

$$\begin{array}{r} 354 \\ +167 \\ \hline 521 \end{array}$$

In order to arrive at the answer we

- Add the digits in the unit's column, $4 + 7 = 11$.
This is 1 ten and 1 unit; record the 1 unit in the unit's place and record 1 ten above the ten's column.
- Add the digits in the ten's column; $1 + 5 + 6 = 12$.
This number represents 2 tens and 1 hundred.
Record the 2 in the ten's place and record the 1 above the hundred's column.
- Add the digits in the hundred's column; $1+3+1=5$.
This number represents 5 hundred. Record the 5 in the hundred's column.

What is going on when you sum two numbers in this way?

The answer lies in reconciling this process with one that implements the commutative and associative laws as they apply to $354 + 167$ expressed in base-10 positional notation. That is, writing the problem as

$$(300 + 50 + 4) + (100 + 60 + 7)$$

the commutative and associative laws enable us to regroup and to obtain

$$\begin{aligned}(300 + 100) + (50 + 60) + (4 + 7) &= 400 + 110 + 11 \\&= 400 + (100 + 10) + (10 + 1) \\&= (400 + 100) + (10 + 10) + 1\end{aligned}$$

Whole Numbers and Integers

Addition and Commutative, Associative, and Distributive Laws or “CAD” Laws

Worksheet

When we add $354 + 167$, the numbers can be expressed in base-10 positional notation.

$$(300 + 50 + 4) + (100 + 60 + 7)$$

Explain how the commutative and associative laws are used as we regroup to obtain the following:

$$\begin{aligned}(300 + 100) + (50 + 60) + (4 + 7) \\= 400 + 110 + 11 \\= 400 + (100 + 10) + (10 + 1) \\= (400 + 100) + (10 + 10) + 1\end{aligned}$$

Activity 1

Use the associative property

$$A + (B + C) = (A + B) + C$$

To show why

$$(a + b) + (c + d) = (a + b + c) + d$$

Hint: Let $A = (a + b)$

Answers to Activity 1

$$(a + b) + (c + d) = (a + b + c) + d$$

Let $A = (a + b)$

$$\begin{aligned}(a + b) + (c + d) &= A + (c + d) && \text{substitution} \\&= (A + c) + d && \text{associative} \\&= (a + b + c) + d && \text{substitution}\end{aligned}$$

How are commutative and associative laws used in the addition algorithm?

$$\begin{array}{r} 52 \\ +34 \\ \hline 86 \end{array}$$

$$\begin{aligned} 52 + 34 &= (50 + 2) + (30 + 4) \\ &= [(50 + 2) + 30] + 4 && \text{associative} \\ &= [50 + (2 + 30)] + 4 && \text{associative} \\ &= [50 + (30 + 2)] + 4 && \text{commutative} \\ &= [(50 + 30) + 2] + 4 && \text{associative} \\ &= (50 + 30) + (2 + 4) && \text{associative} \\ &= 80 + 6 \\ &= 86 \end{aligned}$$

Important: Addition facts for sums to 20 are to be memorized in first grade.

California Mathematics Standard: Number Sense 2.1, Grade 1.

Activity 2

1. Use the standard algorithm for addition, and then place value expansion to justify your results for

$$45 + 76$$

2. Use the commutative property,

$$a + b = b + a$$

to show why $(c + d) + e = e + (c + d)$

3. Use the commutative and associative properties to show that

$$(a + b) + c = (c + a) + b$$

Answers to Activity 2

1.
$$\begin{aligned} 45 + 76 &= (40 + 5) + (70 + 6) \\ &= [(40 + 5) + 70] + 6 && \text{associative} \\ &= [40 + (5 + 70)] + 6 && \text{associative} \\ &= [40 + (70 + 5)] + 6 && \text{commutative} \\ &= [(40 + 70) + 5] + 6 && \text{associative} \\ &= (40 + 70) + (5 + 6) && \text{associative} \\ &= (110) + (11) && \text{addition facts} \\ &= (100 + 10) + (10 + 1) \\ &= 100 + (10 + 10) + 1 && \text{associative} \\ &= 100 + (20) + 1 && \text{addition facts} \\ &= 121 \end{aligned}$$
2.
$$\begin{aligned} (c + d) + e &= e + (c + d) \quad \text{Let } D = c + d \\ (c + d) + e &= D + e && \text{substitution} \\ &= e + D && \text{commutative} \\ &= e + (c + d) && \text{substitution} \end{aligned}$$
3.
$$\begin{aligned} (a + b) + c &= c + (a + b) && \text{commutative} \\ &= (c + a) + b && \text{associative} \end{aligned}$$

Subtraction: In the same way as addition corresponds to the intuitive operation of "putting together," so does subtraction arise as the intuitive inverse operation of "taking away." In this context, one could argue that the reason that

$$12 - 7 = 5 \text{ is because } 12 = 7 + 5.$$

Indeed, this can be used as the **definition** of subtraction. That is

$$a - b = c \text{ if and only if } a = b + c$$

For example, to compute $728 - 253$, we write

$$\begin{array}{r} 728 \\ -253 \\ \hline 475 \end{array}$$

In order to arrive at the correct answer we,

- a. Subtract the digits in the unit's column, $8 - 3 = 5$
Record the 5 in the unit's place.
- b. Subtract the 5 from the 2 in the ten's column by regrouping one of the 7 hundreds to make 12 tens; then subtract $12 - 5 = 7$. Record the 7 in the ten's place.
- c. Subtract the 2 from the 6 that is left over in the hundred's column; $6 - 2 = 4$.

What is going on when you arrive at the correct answer in this way?

$$\begin{aligned} (700 + 20 + 8) - (200 + 50 + 3) &= (700 - 200) + (20 - 50) + (8 - 3) \\ &= (600 - 200) + (120 - 50) + (8 - 3) \\ &= 400 + 70 + 5 = 475 \end{aligned}$$

Zero and Additive Inverses

Zero is the additive identity:

$$0 + a = a$$

$$a + 0 = a$$

for any number a .

Whole Numbers:

The set of whole numbers (W) is the set of counting numbers (i.e., natural numbers) together with zero.

$$W = \{0, 1, 2, 3, \dots\}$$

Additive Inverse and Negative Numbers

Definition: If a is a whole number, $-a$ is defined to be the number satisfying:

$$-a + a = 0$$

and

$$a + -a = 0$$

Example: If $a = 2$, $-a = -2$,

$$2 + -2 = 0.$$

$-0 = 0$ because $0 + 0 = 0$.

With the definition of the additive inverse for every Whole Number, a new set of numbers is formed.

The Set of Integers

$$Z = \{ \dots -3, -2, -1, 0, 1, 2, 3 \dots \}$$

The Integers may be described as the set of Whole Numbers and their Additive Inverses.

SETS OF NUMBERS

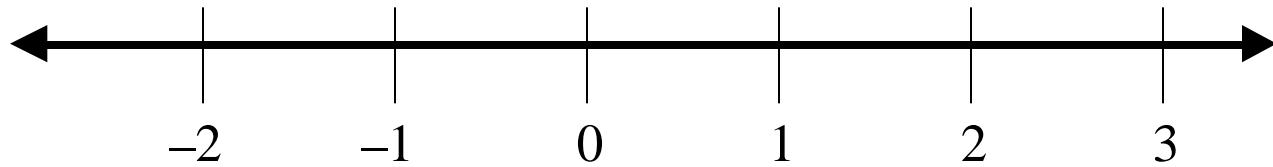
Z = the set of INTEGERS
 $\dots -3, -2, -1$

W = the set of WHOLE NUMBERS
0

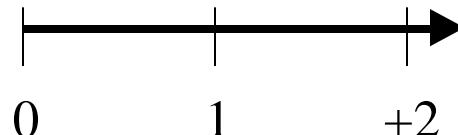
N = the set of NATURAL NUMBERS
1, 2, 3, ...

Question: How are the basic operations of Addition and Subtraction applied to the set of Integers?

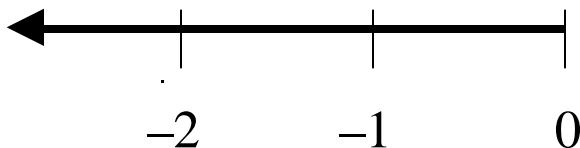
We'll first use a number line to describe Integers



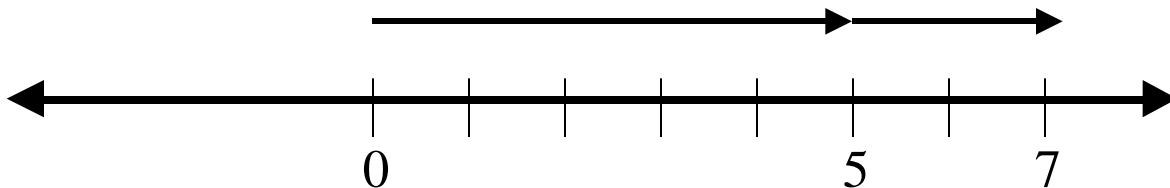
Positive Integers may be represented by arrows that point to the right.



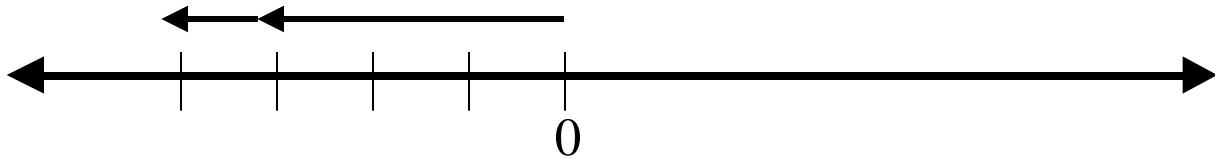
Negative Integers may be represented by arrows that point to the left.



Examples: $5 + 2$

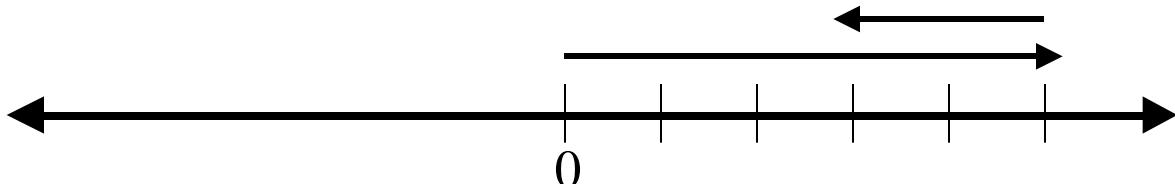


Another example: $-3 + -1$

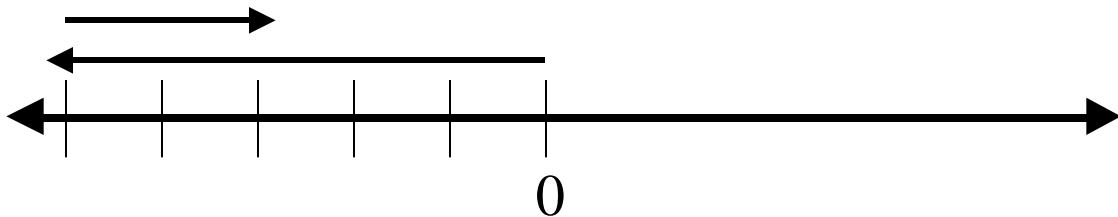


More problems with unlike signs:

Example: $5 + -2$

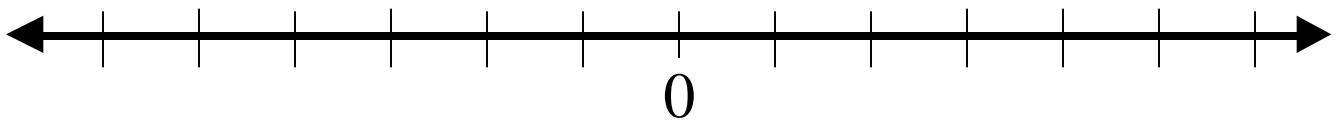


Example: $-5 + 2$

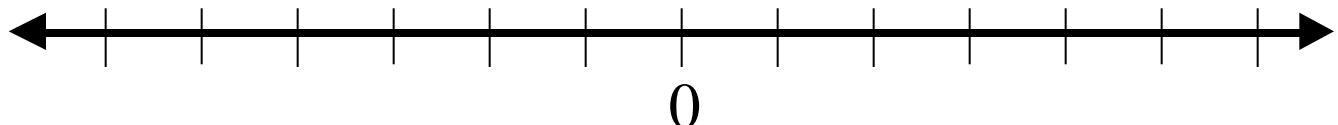


Practice:

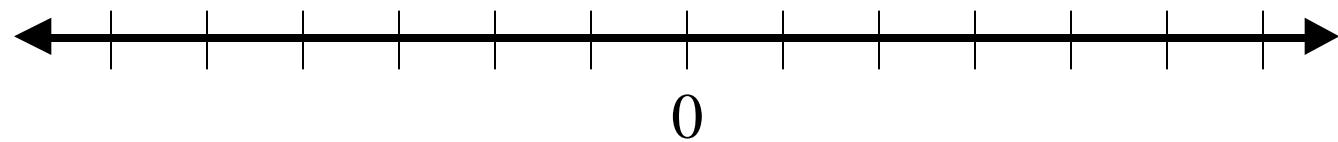
$$2 + 1$$



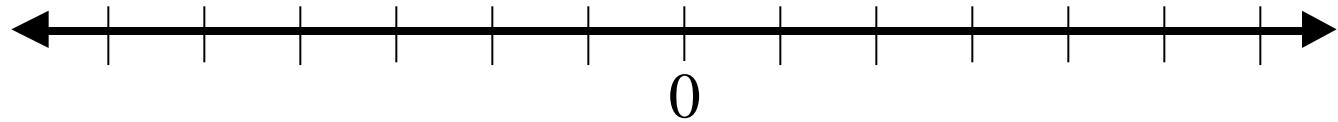
$$-4 + 2$$



$$3 + -4$$



$$-6 + 2$$



The definition of additive inverse for whole numbers can be extended directly to the set of Integers.

Definition: If a is an integer, $-a$ is defined to be the Integer satisfying
And
$$\begin{aligned} -a + a &= 0 \\ a + (-a) &= 0 \end{aligned}$$

Question: What is $-(-7)$?

Using the definition, the additive inverse of (-7) , combined with (-7) must equal 0.

Since $(-7) + 7 = 0$, the additive inverse of (-7) must equal 7, or $-(-7) = 7$.

The Additive Inverse is essential for Subtraction.

Definition: For any two integers a and b ,
$$a - b = a + (-b).$$

Examples:

$$\begin{array}{rclcrcl} 7 - 2 & = & 7 + (-2) & = & 5 \\ 2 - 7 & = & 2 + (-7) & = & -5 \\ 5 - (-3) & = & 5 + 3 & = & 8 \\ -4 - (-2) & = & -4 + 2 & = & -2 \end{array}$$

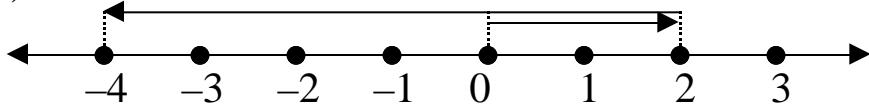
Subtraction has been defined in terms of Addition and the Additive Inverse (Opposite).

Check Your Understanding

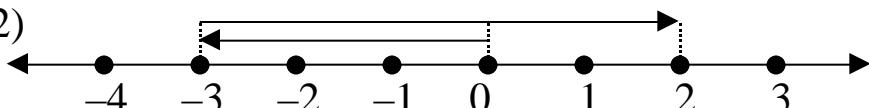
For each “arrow math” problem, write

- (1) an Addition Problem and
- (2) a related Subtraction Problem

1)



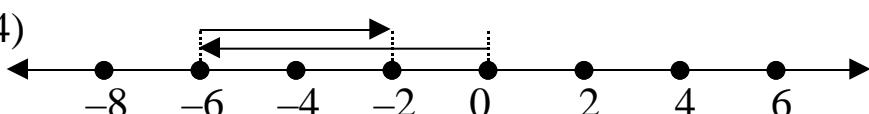
2)



3)

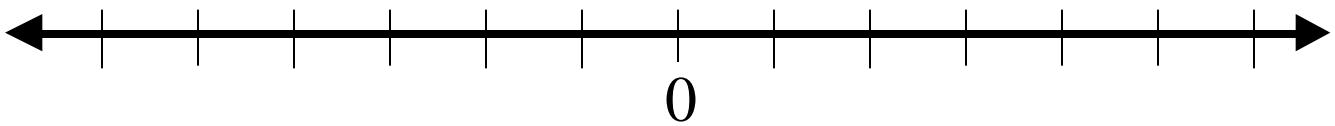


4)

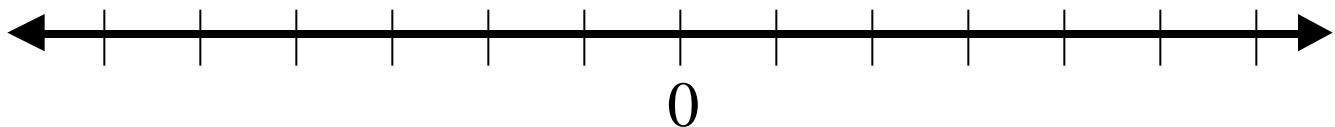
1. (1) _____
(2) _____2. (1) _____
(2) _____3. (1) _____
(2) _____4. (1) _____
(2) _____

Question: Do the Commutative and Associative Laws of Addition hold for the set of Integers?

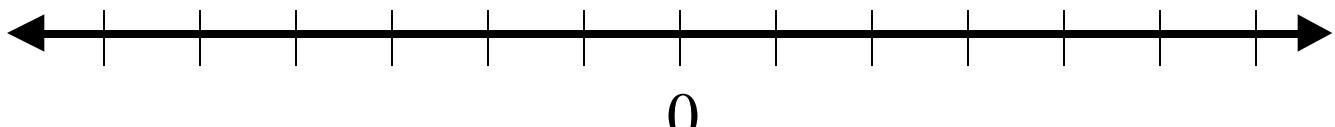
$$4 + -6$$



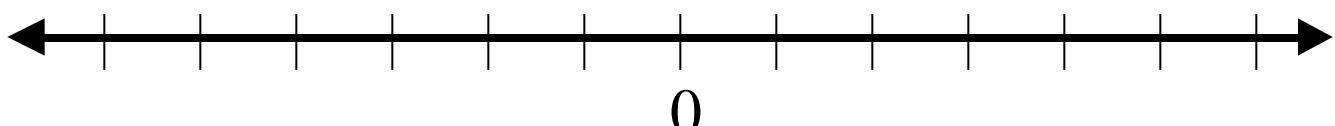
$$-6 + 4$$



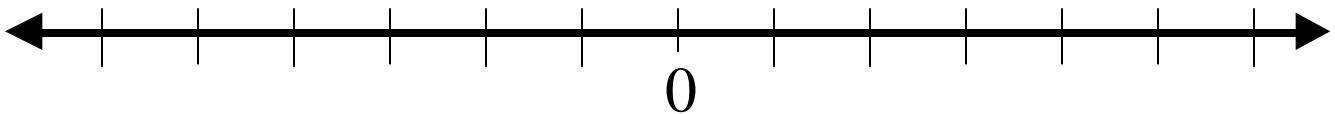
$$-2 + -3$$



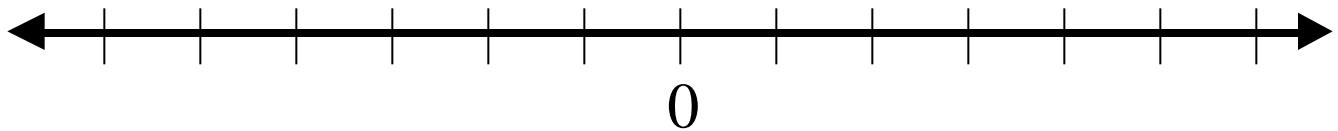
$$-3 + -2$$



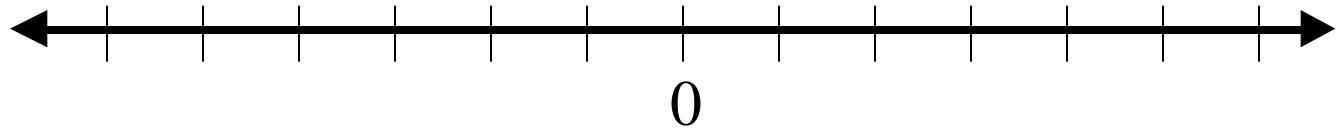
$$(3 + -2) + -4$$



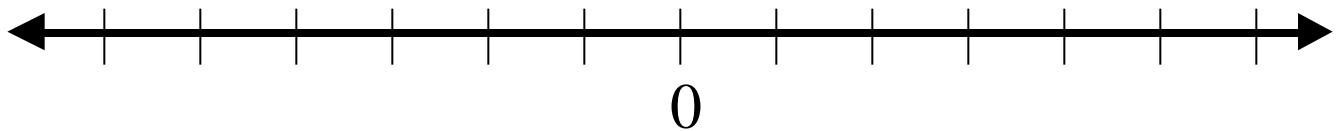
$$3 + (-2 + -4)$$



$$(4 + -6) + 2$$



$$4 + (-6 + 2)$$



A LITTLE ADDITION

$$\begin{array}{r} 8\#3 \\ \&87 \\ + 57\$ \\ \hline \#386 \end{array}$$

In the correctly worked addition problem above, #, & and \$ represent digits. What is the sum of #, & and \$?

- A) 17 B) 18 C) 19 D) 20 E) 21

ANSWER

Since $3 + 7 + \$$ must equal a number with an ending digit of 6, there is only one value of $\$$ that will make it true, so $\$ = 6$ and the sum is 16. For the ten's place a 1 must be added to the sum of $\# + 8 + 7$ and the sum must end with a digit of 8. so $\# + 16$ must end in an 8, the only digit possible is 2. Therefore, $\# = 2$. The hundred's place sum must equal 23 since $\#$ was replaced by a 2. The digits to be added are $8 + \& + 5$ and 1. Again, $14 + \&$ must equal 23, so $\& = 9$.

The sum of $\# + \& + \$ =$ the sum of
 $2 + 9 + 6 = 17$.

The correct answer choice is (A).

Post Test

1. What is the commutative property of addition?
2. What is the associative property of addition?
3. Is subtraction commutative? Explain.
4. Calculate $-2 + 5 - (4 - 7)$.
5. Write 37,205 in expanded form.
6. Explain in detail the addition algorithm for the example
$$\begin{array}{r} 36 \\ +47 \\ \hline 83 \end{array}$$

Post Test Answer Key

1. For any numbers a and b ,

$$a + b = b + a$$

2. For any numbers a , b , and c ,

$$a + (b + c) = (a + b) + c$$

3. No. For example, $3 - 2$ $2 - 3$

4. $-2 + 5 - (4 - 7) = 3 - (4 - 7) = 3 - -3 = 3 + 3 = 6$

5. $37,205 = 3 \cdot 10^4 + 7 \cdot 10^3 + 2 \cdot 10^2 + 0 \cdot 10 + 5$

6. $36 + 47 = (30 + 6) + (40 + 7)$

$$\begin{aligned} &= [(30 + 6) + 40] + 7 && \text{associative} \\ &= [30 + (6 + 40)] + 7 && \text{associative} \\ &= [30 + (40 + 6)] + 7 && \text{commutative} \\ &= [(30 + 40) + 6] + 7 && \text{associative} \\ &= (30 + 40) + (6 + 7) && \text{associative} \\ &= (70) + (13) && \text{addition facts} \\ &= 70 + (10 + 3) \\ &= (70 + 10) + 3 && \text{associative} \\ &= 80 + 3 && \text{addition facts} \\ &= 83 \end{aligned}$$