HW Set on Chapter 5

① Test 543,210,756 for divisibility

by 2 YES last digit is even

by 3 YES sum of digits = 33 and $3|33$

by 4 YES $4|56$ ← number represented by The LAST TWO DIGITS

by 5 NO last digit is 6, not 0 or 5

by 6 YES since divisible by both 2 and 3

by 8 NO $8 \nmid 756$ ← number represented by The LAST THREE DIGITS

by 9 NO $9 \nmid 33$ ← SUM of digits

by 10 NO last digit is not 0

by 11 sum of digits in even powers of 10 spots $= 5+3+1+7+6 = 22$
sum of digits in odd powers of 10 spots $= 4+2+0+5 = 11$

Does $11 | (22-11)$? YES, so YES div by 11

difference of above sums

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② T or F? If F, provide a counter-example

(a) T If $12|a$, then $3|a$ Why? If $12|a$, then $a = 12m$ for some whole number m. So $a = 3 \cdot 4 \cdot m = \underline{3}(4m)$, showing $3|a$
In fact, if $12|a$, then <u>any</u> factor of 12 will divide a
(that is, if $12|a$, then 2, 3, 4, 6 will also divide a)

(b) F If $2|a$, then $4|a$ For example, $2|6$ but $4 \nmid 6$

(c) F If $5|(a+b)$, then $5|a$ or $5|b$
For example $5|(3+7)$ but $5 \nmid 3$ and $5 \nmid 7$
(you have to think of two numbers whose <u>SUM</u> is divisible by 5 but both numbers are <u>not</u> divisible by 5)

(d) T If $5|a$ and $5|b$, then $5|ab$ Why?
We already showed, if $5|a$, then 5 divides any multiple of a
So if $5|a$, then $5|ab$ (it's irrelevant whether or not 5 divides b)

This is a multiple of a (namely $b \cdot a$)

(in fact, if $5|a$ and $5|b$, then $a = 5m$ and $b = 5n$ for some whole numbers m, n So $a \cdot b = 5m \cdot 5n = 25\,mn$ So we can even conclude $25|ab$, which certainly implies $5|ab$)

(e) F If $4|a$ and $8|a$, then $32|a$

for example, $4|40$ and $8|40$, but $32 \nmid 40$
another ex, $4|8$ and $8|8$, but $32 \nmid 8$

(The reason This fails is because 4 and 8 have common factors other Than 1
GCF(4,8) = 4 ($\neq 1$))

③ (a) In the TEST FOR PRIMENESS,
for 259, check all primes p satisfying $p^2 \le 259$
for 263, check all primes p satisfying $p^2 \le 263$

So for both, check 2, 3, 5, 7, 11, 13

$13^2 = 169$ $17^2 \ne 289$ (which is > 259; > 263)

(alternatively $\sqrt{259} \sim 16.1$, $\sqrt{263} \sim 16.2$ So 13 is the largest prime $< \sqrt{259}$; $< \sqrt{263}$)

(b) Check if 259 is divisible by any prime in our list
$2 \nmid 259$, $3 \nmid 259$, $5 \nmid 259$, $7 \mid 259$ STOP! 259 is COMPOSITE (in fact, $259 = 7 \cdot 37$)

(c) Check if 263 is divisible by any prime in our list
$2 \nmid 263$, $3 \nmid 263$, $5 \nmid 263$, $7 \nmid 263$, $11 \nmid 263$, $13 \nmid 263$ So 263 is PRIME

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④ $a = 2^2 \cdot 5 \cdot 11^5 \cdot 13 \cdot 23$
$b = 2^3 \cdot 5^2 \cdot 7^2 \cdot 11^3 \cdot 23$

$GCF(a,b) = 2^2 \cdot 5 \cdot 11^3 \cdot 23$
$LCM(a,b) = 2^3 \cdot 5^2 \cdot 7^2 \cdot 11^5 \cdot 13 \cdot 23$

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⑤ (a) Find GCF(9192, 96) using the EUCLIDEAN ALGORITHM

$$\begin{array}{r} 95 \\ 96\overline{)9192} \\ \underline{864} \\ 552 \\ \underline{480} \\ 72 \end{array} \qquad \begin{array}{r} 1 \\ 72\overline{)96} \\ \underline{72} \\ * \; 24 \end{array} \qquad \begin{array}{r} 3 \\ 24\overline{)72} \\ \underline{72} \\ 0 \end{array}$$

LAST NONZERO REMAINDER IS THE GCF

$GCF(9192, 96) = GCF(96, 72)$
$= GCF(72, 24)$
$= GCF(24, 0)$
$= 24$

(b) $GCF(a,b) \cdot LCM(a,b) = a \cdot b$
So $LCM(a,b) = \dfrac{a \cdot b}{GCF(a,b)}$

So $LCM(9192, 96) = \dfrac{(9192)(96)}{\underbrace{24}_{GCF(9192,96)}}$
$= 36{,}768$

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⑥ Must find LCM (42, 54)
$42 = 2 \cdot 3 \cdot 7$
$54 = 2 \cdot 3^3$
So $LCM(42, 54) = 2 \cdot 3^3 \cdot 7 = 378$ minutes
$= 6$ hrs 18 mins

$$\begin{array}{r} 6 \\ 60\overline{)378} \\ \underline{360} \\ 18 \end{array}$$

The first time after 8:00 am that planes will leave simultaneously for S.F. is $\boxed{\text{2:18 pm}}$ (that is, 6 hrs 18 mins after 8 am)

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⑦ Here we are dividing into 90, 120, 225. The number of bags is a common divisor. The greatest number of bags he could use is given by the GCF(90, 120, 225)

$90 = 2 \cdot 3^2 \cdot 5$
$120 = 2^3 \cdot 3 \cdot 5$
$225 = 3^2 \cdot 5^2$

$GCF(90, 120, 225) = 3 \cdot 5 = \boxed{\text{15 bags}}$ *

Note: $15\overline{)90}$ 6 blue in each bag $15\overline{)120}$ 8 green in each bag $15\overline{)225}$ 15 white in each bag

So 6 + 8 + 15 or 29 marbles in each of the 15 bags