Equivalent Fractions

When do two fractions represent the same amount? Or, when do they represent the same point or number on a number line?

Fold a piece of paper in half:

We now have 2 equal regions:

The shaded part represents \( \frac{1}{2} \) of the paper.

Fold again, the same way.

We now have four equal regions:

the shaded part remains the same, but now represents \( \frac{2}{4} \) of the paper.

Both representations are the same part of the whole paper.

\[
\frac{1}{2} = \frac{2}{4}
\]
More Equivalent Fractions

Now fold the paper across.
We now have 8 regions of the paper, and the shaded region now represents \( \frac{4}{8} \) of the paper.

Notice that \( \frac{1}{2} \) can be written in more than one way.

\[
\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \text{etc.}
\]

All of these fractions are located at the same point on the number line. Therefore, they represent the same number.
Paper Folding to Symbols

From the paper folding exercises we see that increasing both numerator and denominator of a fraction by the same factor gives an equivalent or equal fraction.

It is a crucial step to move toward the use of mathematical symbols to express these ideas....
More Equivalent Fractions

In symbolic form,

\[
\frac{2}{3} \text{ can be rewritten as } \frac{2\cdot k}{3\cdot k} \text{ for any integer } k.
\]

\[
\begin{align*}
\frac{2}{3} &= \frac{2\cdot 2}{3\cdot 2} = \frac{4}{6} \\
\frac{1}{3} &= \frac{1\cdot 2}{3\cdot 2} = \frac{2}{6} \\
\frac{2}{3} &= \frac{2\cdot 3}{3\cdot 3} = \frac{6}{9} \\
\frac{1}{3} &= \frac{1\cdot 3}{3\cdot 3} = \frac{3}{9} \\
\frac{2}{3} &= \frac{2\cdot 4}{3\cdot 4} = \frac{8}{12} \\
\frac{1}{3} &= \frac{1\cdot 4}{3\cdot 4} = \frac{4}{12}
\end{align*}
\]

Leading to:

\[
\begin{align*}
\frac{2}{3} &= \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \text{ etc.} \\
\frac{1}{3} &= \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \text{ etc.}
\end{align*}
\]
More Equivalent Fractions

The numerator and denominator can be divided, or multiplied, by the same factor to get an equivalent fraction.

\[
\frac{6}{9} = \frac{6 \div 3}{9 \div 3} = \frac{2}{3}
\]

\[
\frac{12}{15} = \frac{12 \div 3}{15 \div 3} = \frac{4}{5}
\]

\[
\frac{1}{10} = \frac{1 \cdot 100}{10 \cdot 100} = \frac{100}{1,000}
\]

\[
\frac{4}{9} = \frac{4 \cdot 4}{9 \cdot 4} = \frac{16}{36}
\]

But not all equivalent fractions are related in this way. For example:
More Equivalent Fractions

\[ \frac{3}{6} \text{ and } \frac{4}{8} \text{ each equal } \frac{1}{2}, \]

But no whole number multiple of 3 gives 4 and

no whole number multiple of 6 gives 8

Similarly

\[ \frac{91}{119} = \frac{143}{187} \]

because they both reduce to \( \frac{13}{17} \).

But it is not easy to see that they are equal.
Question

Is there a simple way to tell when any two fractions are equal?

Yes! It's called "cross multiplication."

Two fractions \(\frac{a}{b}\) and \(\frac{c}{d}\) are equal if and only if

\[ad = bc.\]

For example,

\[
\frac{3}{6} = \frac{4}{8} \text{ because } 3 \cdot 8 = 6 \cdot 4
\]

\[
\frac{91}{119} = \frac{143}{187} \text{ because } 91 \cdot 187 = 119 \cdot 143
\]

But why does this rule work?
Explanation for Cross Multiplication

To determine whether

\[
\frac{a}{b} = \frac{c}{d}
\]

find a common denominator for each fraction.

\[
b d = 6 \cdot 8
\]

is a common denominator for both

\[
\frac{a}{b} \quad \text{and} \quad \frac{c}{d}
\]

Multiply:

numerator and denominator of \( \frac{a}{b} \) by \( d \)

\[
\frac{a}{b} = \frac{ad}{bd}
\]

\[
\frac{3}{6} \quad \text{by} \quad 8
\]

numerator and denominator of \( \frac{c}{d} \) by \( b \)

\[
\frac{c}{d} = \frac{bc}{bd}
\]

\[
\frac{4}{8} \quad \text{by} \quad 6
\]

when \( ad = bc \),

\[
(3 \cdot 8) = (4 \cdot 6)
\]

the two fractions are equal.
Cross Multiplication

Perspective: different fractions like \( \frac{1}{2} \) and \( \frac{2}{4} \) represent the same point on a number line and therefore, they represent the same **number**. We need a simple criterion to decide when two fractions represent the same number.

**Definition 2** Two fractions \( \frac{a}{b} \) and \( \frac{c}{d} \) are equal, that is, they represent the same number, if and only if

\[
ad = bc
\]
Ordering Fractions

The explanation leading to cross multiplication shows a way to compare unequal fractions.

Proposition: \( \frac{a}{b} < \frac{c}{d} \) if and only if \( ad < bc \).

Is \( \frac{4}{7} \) larger than \( \frac{3}{5} \)?

Since \( 4 \cdot 5 < 7 \cdot 3 \) then \( \frac{4}{7} < \frac{3}{5} \)

Is \( \frac{2}{3} < \frac{3}{4} \)?

since \( 2 \cdot 4 < 3 \cdot 3 \)

\( 8 < 9 \)

\( \frac{2}{3} < \frac{3}{4} \)

Alternatively,

\( \frac{2}{3} = \frac{8}{12} \) and \( \frac{3}{4} = \frac{9}{12} \)

Therefore, \( \frac{2}{3} < \frac{3}{4} \).
Determine Which Fraction is Larger

Worksheet

Which rational number is larger?

\[ \frac{2}{3} \text{ or } \frac{3}{4} ? \]

\[ \frac{3}{10} \text{ or } \frac{3}{9} ? \]

\[ 1\frac{3}{4} \text{ or } 1\frac{5}{8} ? \]
More Equivalent Fractions

*Definition 2* includes the cases where some or all of $a, b, c,$ and $d$ are negative.

\[
\frac{3}{-4} = \frac{-3}{4}
\]

using cross multiplication

\[
3 \cdot 4 = (-3) \cdot (-4)
\]
\[
12 = 12
\]

or by multiplying the top and bottom of one fraction by -1

\[
\frac{3}{-4} = \frac{3 \cdot (-1)}{-4 \cdot (-1)} = \frac{-3}{4}
\]
More Equivalent Fractions

Another example

\[
\frac{3}{4} = \frac{-3}{-4}
\]

using cross multiplication

\[
3 \cdot (-4) = (-3) \cdot 4
\]

\[-12 = -12\]

or by multiplying the top and bottom of one fraction by -1

\[
\frac{3}{4} = \frac{3 \cdot (-1)}{4 \cdot (-1)} = \frac{-3}{-4}
\]
Fractions are Rational Numbers

*Definition 3:* A rational number is any number which can be expressed in the form \( \frac{a}{b} \) where \( a \) and \( b \) are integers; \( b \) does not equal zero.

**Fractions represent rational numbers.**

More than one fraction represents the same rational number,

\[
\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \text{etc.}
\]

Rational numbers may be thought of as points on the number line and fractions are the names we call them.
SETS of NUMBERS

Rational Numbers

any number that can be expressed in the form of \( \frac{a}{b} \), \( b \neq 0 \), where \( a \) and \( b \) are integers.

Integers

… -3, -2, -1, 0, 1, 2, 3...

Whole Numbers
0

Counting Numbers
1, 2, 3, …
Addition and Subtraction of Fractions

Case 1

Two fractions which are to be added or subtracted with the same denominator.

\[
\frac{1}{3} + \frac{1}{3} = \frac{1+1}{3} = \frac{2}{3} \quad \frac{4}{5} - \frac{1}{5} = \frac{4-1}{5} = \frac{3}{5}
\]

In general

\[
\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b} \quad \frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}
\]
Adding and Subtracting Fractions

Case 2
Two fractions which are to be added or subtracted with different denominators.

An important formula for all cases:

\[
\frac{a}{b} + \frac{c}{d} = \frac{(ad + bc)}{bd}
\]

Justification:

\[
\frac{a}{b} + \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{d} + \frac{c}{d} \cdot \frac{b}{b}
\]

\[
= \left( \frac{a \cdot d}{b \cdot d} \right) + \left( \frac{c \cdot b}{d \cdot b} \right)
\]

\[
= \frac{ad}{bd} + \frac{cb}{db}
\]

\[
= \frac{(ad + bc)}{bd}
\]

Therefore:

\[
\frac{a}{b} + \frac{c}{d} = \frac{(ad + bc)}{bd}
\]

This formula is important in algebra.
Examples

\[ \frac{2}{3} + \frac{4}{5} = \frac{2 \cdot 5}{3 \cdot 5} + \frac{4 \cdot 3}{5 \cdot 3} = \frac{2 \cdot 5 + 3 \cdot 4}{3 \cdot 5} \]

\[ = \frac{10 + 12}{15} \]

\[ = \frac{22}{15} \]

\[ = 1 \frac{7}{15} \]

\[ \frac{1}{3} + \frac{1}{6} = \frac{6 \cdot 1 + 3 \cdot 1}{3 \cdot 6} \]

\[ = \frac{9}{18} = \frac{1}{2} \]

In practice, common denominators less than \(bd\) are used. The lowest common denominator for the fractions \(\frac{a}{b}\) and \(\frac{c}{d}\) is the Least Common Multiple (LCM) of \(b\) and \(d\).
More Equivalent Fractions

To justify \(-\frac{3}{4} = \frac{-3}{4}\)

\[
\frac{-3}{4} + \frac{3}{4} = \frac{-3 + 3}{4} = \frac{0}{4} = 0
\]

and

\[
-\frac{3}{4} + \frac{3}{4} = 0
\]

because any number added to its opposite = 0

therefore \(-\frac{3}{4} = \frac{-3}{4}\)
Mixed Numbers to Improper Fractions

An important definition to remember is:

\[ 5 \frac{2}{3} = 5 + \frac{2}{3} \]

Converting the mixed number to an improper fraction can be done using addition of fractions.

\[ 5 \frac{2}{3} = 5 + \frac{2}{3} = \frac{15}{3} + \frac{2}{3} = \frac{17}{3} \]

A shortcut to convert \( \frac{a}{b} \) to an improper fraction is to multiply the whole number (w) times the denominator (b) and then add the numerator (a).

That is \( \frac{a}{b} = \frac{wb + a}{b} \)

Example: \( 5 \frac{2}{3} = \frac{17}{3} \)

because \( 5 \cdot 3 + 2 = 17 \)
Converting a Mixed Number to an Improper Fraction Worksheet

Explain why these two ways of converting a mixed number to an improper fraction are really the same.

By using addition of fractions we get \( \frac{5\frac{2}{3}}{=} 5 + \frac{2}{3} = \frac{17}{3} \), or by multiplying the whole number times the denominator and then adding the numerator we get \( 5\frac{2}{3} = \frac{17}{3} \).
Improper Fractions to Mixed Numbers

\[ \frac{17}{3} = \frac{15}{3} + \frac{2}{3} = 5 \frac{2}{3} \]

Long division tells us how to do this

\[ 3 \overline{)17} \text{ gives} \]

5 with remainder 2

or

\[ 17 = 3 \cdot 5 + 2 \]

So

\[ \frac{17}{3} = \frac{3 \cdot 5 + 2}{3} = \frac{5 \cdot 3}{3} + \frac{2}{3} \]

\[ = 5 + \frac{2}{3} \]

\[ = 5 \frac{2}{3} \]
Multiplication of Fractions

Definition 4: The product of $\frac{a}{b}$ and $\frac{c}{d}$

for any $a$, $b$, $c$, and $d$ (with denominators not zero) is given by

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

Goal: Explain why this is a good definition.

Case 1: an integer times a unit fraction.

$$4 \cdot \frac{1}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$$

$$= \frac{(1+1+1+1)}{5}$$

$$= \frac{4}{5}$$

So, $4 \cdot \frac{1}{5} = \frac{4}{5}$

$$4 = \frac{4}{1} \quad \text{so,} \quad \frac{4}{1} \cdot \frac{1}{5} = \frac{4}{5}$$

This matches Definition 4.
Another way to look at it:

\[ 4 \cdot \frac{1}{5} = \frac{1}{5} \cdot 4 \]

or

\[ \frac{1}{5} \text{ of } 4 \]

or

\[ 4 \div 5 \]

The multiplication symbol “•” represents the word “of” in word problems. So,

\[ \frac{1}{5} \cdot 4 = \frac{1}{5} \text{ of } 4 = 4 \div 5. \]

Since

\[ \frac{1}{5} \cdot 4 = \frac{4}{5} \]

the fraction \( \frac{4}{5} \) may be thought of as “4 ÷ 5.”
Fractions may be thought of as the “answers” to division problems which have no answer if you only had whole numbers or integers.

Recall that using integers, $25 ÷ 6$ has the answer $q = 4$ and $r = 1$.

With fractions, division of whole numbers no longer need remainders.

Using fractions $25 ÷ 6$ simply becomes $\frac{25}{6}$ or $4\frac{1}{6}$. 
Justification for the Multiplication Formula

Case 2: The product of two unit fractions \( \frac{1}{4} \cdot \frac{1}{3} \)

A one unit square is divided into 4 sections horizontally and 3 sections vertically.

![Diagram of a one unit square divided into 4 sections horizontally and 3 sections vertically, highlighting a subrectangle of dimensions \( \frac{1}{4} \times \frac{1}{3} \).]

One subrectangle has dimensions \( \frac{1}{4} \) by \( \frac{1}{3} \).

The area of one subrectangle is \( \frac{1}{4} \cdot \frac{1}{3} \).

There are \( 3 \cdot 4 = 12 \) subrectangles in this unit square, each with the same area.

Area of one subrectangle is the area of the square (1 x 1) divided by 12, or \( \frac{1}{12} \). So, \( \frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12} \).

This is consistent with the formula \( \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \).
Last Step in Justifying Multiplication Formula

We are ready to deduce the formula \[ \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \]

**Case 3**: The general case.
Using the associative and commutative properties of multiplication

\[
\frac{a}{b} \cdot \frac{c}{d} = \left( \frac{a}{b} \cdot 1 \right) \cdot \left( 1 \cdot \frac{c}{d} \right) \quad \text{Case 1}
\]

\[
= a \cdot \left( \frac{1}{b} \cdot \frac{1}{d} \right) \cdot c \quad \text{Associative Property}
\]

\[
= \left( a \cdot c \right) \cdot \left( \frac{1}{b} \cdot \frac{1}{d} \right) \quad \text{Communicative & Associative Prop.}
\]

\[
= \left( a \cdot c \right) \cdot \frac{1}{bd} \quad \text{Case 2}
\]

\[
= \frac{ac}{bd} \quad \text{Case 1}
\]

Both **Case 1** and **Case 2** have been used.
Example: \[
\frac{3}{4} \times \frac{5}{7} = \frac{3 \times 5}{4 \times 7} = \frac{15}{28}
\]

Here are the steps we just used for Case 3:

\[
\frac{3}{4} \times \frac{5}{7} = (3 \times \frac{1}{4}) \times (\frac{1}{7} \times 5)
\]

\[
= 3 \times (\frac{1}{4} \times \frac{1}{7}) \times 5
\]

\[
= (3 \times 5)(\frac{1}{4} \times \frac{1}{7})
\]

\[
= (15)(\frac{1}{28})
\]

\[
= \frac{15}{28}
\]
Multiplication of Fractions
Worksheet

1. \( \frac{5}{6} \cdot \frac{2}{3} = \)

2. \( \frac{5}{8} \cdot \frac{7}{9} = \)

3. \( 1 \frac{2}{3} \cdot \frac{7}{10} = \)

4. \( 2 \frac{3}{4} \cdot 5 \frac{1}{6} = \)
Division of Fractions

Definition 5: For any fractions \( \frac{a}{b} \) and \( \frac{c}{d} \),

\[
\frac{a}{b} \div \frac{c}{d} = \frac{a \cdot d}{b \cdot c} = \frac{ad}{bc}
\]

An example:

\[
\frac{3}{4} \div \frac{7}{3} = \frac{3}{4} \cdot \frac{3}{7} = \frac{9}{28}
\]

Why is this the right way to divide fractions?
Why do you invert and multiply?
Division of Fractions

One way to understand division of fractions is by recognizing division as the inverse of multiplication.

\[ A \div B = C \]

means the same as

\[ C \cdot B = A \]

For example, \( 12 \div 4 = 3 \)

because \( 3 \cdot 4 = 12 \)

With fractions:

\[ \frac{a}{b} \div \frac{c}{d} = \frac{e}{f} \]

means the same as

\[ \frac{e}{f} \cdot \frac{c}{d} = \frac{a}{b} \]
Division of Fractions

Solve for $\frac{e}{f}$ by multiplying both sides by $\frac{d}{c}$:

$$\left(\frac{e}{f} \cdot \frac{c}{d}\right) \cdot \frac{d}{c} = \frac{a}{b} \cdot \frac{d}{c}$$

$$\frac{e}{f} \cdot \left(\frac{c}{d} \cdot \frac{d}{c}\right) = \frac{a}{b} \cdot \frac{d}{c}$$

$$\frac{e}{f} \cdot \frac{cd}{cd} = \frac{a}{b} \cdot \frac{d}{c}$$

$$\frac{e}{f} \cdot 1 = \frac{a}{b} \cdot \frac{d}{c}$$

$$\frac{e}{f} = \frac{a}{b} \cdot \frac{d}{c}$$

Since

$$\frac{e}{f} = \frac{a}{b} \div \frac{c}{d}$$

It follows that

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

To understand more clearly this explanation, let's try it with actual numbers…
Division of Fractions

\[
\frac{2}{3} \div \frac{5}{7} = \frac{14}{15}
\]

means the same as

\[
\frac{14}{15} \cdot \frac{5}{7} = \frac{2}{3}
\]

Solve for \(\frac{14}{15}\) by multiplying both sides by \(\frac{7}{5}\):

\[
\left( \frac{14}{15} \cdot \frac{5}{7} \right) \cdot \frac{7}{5} = \frac{2}{3} \cdot \frac{7}{5}
\]

\[
\frac{14}{15} \cdot \left( \frac{5}{7} \cdot \frac{7}{5} \right) = \frac{2}{3} \cdot \frac{7}{5}
\]

\[
\frac{14}{15} \cdot \frac{35}{35} = \frac{2}{3} \cdot \frac{7}{5}
\]

\[
\frac{14}{15} \cdot 1 = \frac{2}{3} \cdot \frac{7}{5}
\]

Since

\[
\frac{14}{15} = \frac{2}{3} \cdot \frac{5}{7}
\]

Therefore

\[
\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \cdot \frac{7}{5}
\]
More Insight into Division of Fractions

\[ 1 \div \frac{1}{4} = \]

means

how many one-fourths are in 1 whole?

Each subrectangle is one-fourth of the unit square.
There are 4 of the one-quarter units in the unit square.

So: \[ 1 \div \frac{1}{4} = 1 \cdot \frac{4}{1} = 4 \]
Dividing Fractions by  
The Common Denominators Approach

Think of

\[
\frac{6}{7} \div \frac{2}{7}
\]

as 6 groups of \(\frac{1}{7}\) of something divided by 2 groups of \(\frac{1}{7}\) of that thing. This is like dividing 6 apples into groups of 2 apples.

Dividing fractions with the same denominator can be done by just dividing the numerators.
\[
\frac{6}{7} \div \frac{2}{7} = 3
\]
Another Way to Understand Division of Fractions

To find \( \frac{a}{b} \div \frac{c}{d} \)

first find a common denominator for both fractions = \(bd\)

Rewrite \( \frac{a}{b} \) and \( \frac{c}{d} \) as:

\[
\frac{a}{b} = \frac{ad}{bd} \quad \text{and} \quad \frac{c}{d} = \frac{bc}{bd}
\]

Then

\[
\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bd} \div \frac{bc}{bd}
\]

Since these last two fractions have the same denominator, just divide the numerators

So, the answer is \(ad \div bc\) or \(\frac{ad}{bc}\).

Therefore,

\[
\frac{a}{b} \div \frac{c}{d} = \frac{a \cdot d}{b \cdot c}
\]
Another Explanation for Division of Fractions

\[
\frac{1}{3} \div \frac{2}{5} = \]

may be written: \[
\frac{3}{2} = \]

and multiplying this by 1 we get:

\[
\frac{1}{3} \cdot \frac{5}{2} = \frac{3}{2} \cdot \frac{2}{5} = \frac{3}{2} \cdot \frac{2}{5}
\]

\[
\frac{5}{6} = \frac{6}{10} = \frac{5}{6}
\]
Another Explanation for Division of Fractions

\[
\frac{a}{b} \div \frac{c}{d}
\]

May also be written: \(\frac{a}{b} \cdot \frac{c}{d}\)

And multiplying this by 1, we get:

\[
\frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}
\]

\[
= \frac{ad}{bc} = \frac{ad}{bc} \cdot \frac{1}{1} = \frac{ad}{bc}
\]

So,

\[
\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}
\]
A special note:

Many students through the years complain about not understanding fractions. They will often avoid problems involving fractions. Traditional sequence of adding, subtracting, multiplying, and dividing natural numbers, whole numbers, and integers leads us to do the same with rational numbers. However, when we get to Algebra, the order is often reversed when working with polynomials involving rational expressions.

Teachers might give students greater security when working with fractions by capitalizing on their successes. Students tend to find reducing of fractions an easier task than finding common denominators when adding or subtracting them. Some students may find greater success with fractions by multiplying and dividing them first and then getting to one of the most difficult concepts to learn; addition and subtraction of fractions with unlike denominators.

Some teachers may wish to teach addition and subtraction of fractions with Case 1 conditions, move to multiplication and division of fractions as an extension of reducing or building fractions, and then conclude with adding and subtracting fractions with unlike denominators as in Case 2 conditions when they are more experienced in working with fractions.
A Problem with the Jumbo Inch

C \cdot D = ?

A Problem with the Jumbo Inch

\[
\begin{array}{cccccc}
& & & & & \\
A & 0 & B & C & D & 1 \\
& & & & & E
\end{array}
\]

\[C \cdot D = ?\]

\[\begin{array}{lllll}
\end{array}\]

The approach to solving this problem is to assign values to C and D. For example, \(C = \frac{1}{2}\) and \(D = \frac{4}{5}\), then \(C \cdot D = \frac{1}{2} \cdot \frac{4}{5} = \frac{4}{10} = \frac{2}{5}\). Since \(\frac{2}{5} < \frac{1}{2}\) (C) and \(\frac{2}{5} < \frac{4}{5}\) (D), the answer could be A and B; but since the fractions are positive, the result of A could never be obtained, and the answer is B.
Optional Word Problems

1. A box of laundry detergent contains 40 cups. If your washing machine takes $1 \frac{1}{4}$ cups per load, how many loads of wash can you do?

2. Sandra, her brother, and another partner own a restaurant. If Sandra owns $\frac{1}{3}$ and her brother owns $\frac{2}{7}$, what part does the third partner own?

Answers:

1. $40 \div 1 \frac{1}{4} = 40 \div \frac{5}{4} = 40 \times \frac{4}{5} = 8 \times 4 = 32$ loads.

2. $1 - \left( \frac{1}{3} + \frac{2}{7} \right) = 1 - \left( \frac{7}{21} + \frac{6}{21} \right) = 1 - \frac{13}{21} = \frac{21}{21} - \frac{13}{21} = \frac{8}{21}$ of the restaurant.
Post Test

1. \( \frac{1}{5} + \frac{2}{5} = \)

2. \( \frac{3}{4} \cdot \frac{1}{3} = \)

3. \( \frac{3}{12} - \frac{2}{15} = \)

4. \( \frac{25}{12} \div \frac{3}{4} = \)

5. \( 124 \div 3 \frac{1}{2} = \)

6. Express \( \frac{5}{a} \cdot \frac{b}{9} \) as a single fraction in terms of \( a \) and \( b \).

7. Express \( \frac{c}{4} \div \frac{5}{d} \) as a single fraction in terms of \( c \) and \( d \).

8. Express \( \frac{2}{b} + \frac{c}{7} \) as a single fraction in terms of \( b \) and \( c \).

9. Express \( \frac{a}{3} + \frac{5}{b} \) as a single fraction in terms of \( a \) and \( b \).

10. Convert the following from improper fractions to mixed numbers or vice versa:
    
    a. \( \frac{3}{4} = \)
    
    b. \( \frac{5}{12} = \)
    
    c. \( \frac{7}{2} = \)
    
    d. \( \frac{25}{3} = \)

10. Which of the following is the larger rational number?
    
    a. \( \frac{3}{4} \) or \( \frac{11}{16} \)?
    
    b. \( \frac{3}{8} \) or \( \frac{5}{12} \)?

11. Define the set of rational numbers.
Post Test Answer Key

1. \( \frac{3}{5} \)  
2. \( \frac{1}{4} \)  
3. \( 1\frac{3}{20} \)  
4. \( \frac{25}{9} \)  
5. \( 35\frac{3}{7} \)  
6. \( \frac{5b}{9a} \)  
7. \( \frac{cd}{20} \)  
8. \( \frac{14 + bc}{7b} \)  
9. \( \frac{ab + 15}{3b} \)  
10. a. \( \frac{7}{4} = \)  
b. \( \frac{41}{12} = \)  
c. \( 3\frac{1}{2} = \)  
d. \( 8\frac{1}{3} = \)  

11. Which of the following is the larger rational number?
   
a. \( \frac{3}{4} > \frac{11}{16} \)  
b. \( 2\frac{3}{8} < \frac{5}{12} \)  

12. The set of rational numbers is the set of all numbers that can be expressed in the form \( \frac{a}{b} \), where \( a \) and \( b \) are integers, and \( b \neq 0 \).