1) a) Find a common multiple of 15 and 12
   The multiples of 15 are 15, 30, 45, 60, ... 
   60 is also a multiple of 12. So 60 is a common multiple of 15 and 12

b) (4A, Practice 18, p 27)

<table>
<thead>
<tr>
<th>Problem</th>
<th>Factors of 18:</th>
<th>1, 2, 3, 6, 9, 18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem</td>
<td>a) factors of 8:</td>
<td>1, 2, 4, 8</td>
</tr>
<tr>
<td></td>
<td>b) factors of 15:</td>
<td>1, 3, 5, 15</td>
</tr>
<tr>
<td></td>
<td>c) factors of 20:</td>
<td>1, 2, 4, 5, 10, 20</td>
</tr>
<tr>
<td></td>
<td>d) factors of 50:</td>
<td>1, 2, 5, 10, 25, 50</td>
</tr>
<tr>
<td>Problem</td>
<td>e) factors of 75:</td>
<td>1, 3, 5, 15, 25, 75</td>
</tr>
<tr>
<td></td>
<td>f) factors of 98:</td>
<td>1, 2, 7, 14, 49, 98</td>
</tr>
</tbody>
</table>

Problem 5 (find a common factor different from 1)

| a) 3 is a common factor of 15 and 6 |
| b) 2 and 4 are common factors of 12 and 16 |
| c) 3 is a common factor of 15 and 18 |

Problem 6

| a) multiples of 2: | 2, 4, 6, 8, ... |
| b) multiples of 6: | 6, 12, 18, 24, ... |
| c) multiples of 8: | 8, 16, 24, 32, ... |

Problem 7

a) The multiples of 4 are 4, 8, 12, ... 
   12 is also a multiple of 3. So 12 is a common multiple of 3 and 4
b) The multiples of 5 are 5, 10, 15, 20, ... 
   20 is also a multiple of 4. So 20 is a common multiple of 4 and 5

c) 12 is a common multiple of 4 and 6

2) Prove that if \( b \) is a multiple of \( a \) (that is, \( a \mid b \)), then \( \text{GCF}(a, b) = a \)

First think of a specific example with one number a multiple of the other to see what the statement is saying; for \( 2 \times 3 \), \( \text{GCF}(3, 27) = 3 \) (and \( \text{LCM}(3, 27) = 27 \))

\[ a \] is the greatest number that divides \( a \). Since we are given \( a \mid b \), \( a \) must be the greatest divisor \( a \) and \( b \) have in common; that is, \( \text{GCF}(a, b) = a \)

Alternatively: Since \( \text{GCF}(a, b) \) must divide \( a \), we must have \( \text{GCF}(a, b) \leq a \) We are given \( a \mid b \). Since \( a \mid a \), \( a \) is a common factor of \( a \) and \( b \). Since \( \text{GCF}(a, b) \) is the greatest common factor of \( a \) and \( b \), \( \text{GCF}(a, b) \geq a \). Putting these together, \( \text{GCF}(a, b) = a \)

Note: If \( a \mid b \), we can also conclude \( \text{LCM}(a, b) = b \)

\( b \) is the smallest (pos) multiple of \( b \). Since we are given \( a \mid b \), \( b \) is a multiple of \( a \). So \( b \) must be the smallest multiple \( a \) and \( b \) have in common; that is, \( \text{LCM}(a, b) = b \)
3) Prove that if \( p \) is prime, then \( GCF(p, a) = 1 \) unless \( a \) is a multiple of \( p \).

The only divisors of a prime \( p \) are 1 and \( p \). If \( p | a \) (that is, \( a \) is a multiple of \( p \)), then \( p \) is the greatest divisor \( p \) and \( a \) have in common. So \( GCF(p, a) = p \) in this case.

If \( p \nmid a \), then the only divisor \( p \) and \( a \) have in common is 1.

So \( GCF(p, a) = 1 \) in this case.

4) a) \( 28 = 2^2 \cdot 7 \)
   \( 63 = 3^2 \cdot 7 \)
   So \( GCF(28, 63) = 7 \)

b) \( 104 = 2^3 \cdot 13 \)
   \( 132 = 2^2 \cdot 3 
   
   So \ GCF(104, 132) = 2^2 = 4 \)

9) \( 24 = 2^3 \cdot 3 \)
   \( 56 = 2^3 \cdot 7 \)
   \( 180 = 2^2 \cdot 3^2 \cdot 5 \)
   \( GCF(24, 56, 180) = 2^2 = 4 \)

5) Euclid's Algorithm

a) \( GCF(91, 52) = GCF(52, 39) = GCF(39, 13) = GCF(13, 0) = 13 \)

\[
\begin{array}{cccc}
52 & 91 & 1 & 13 \\
52 & 91 & 52 & 39 \\
39 & 52 & 39 & 13 \\
13 & 39 & 13 & 0 \\
\end{array}
\]

Last nonzero remainder turns out to be the GCF

b) \( GCF(812, 336) = GCF(336, 140) = GCF(140, 56) = GCF(56, 28) = GCF(28, 0) = 28 \)

\[
\begin{array}{cccc}
812 & 336 & 2 & 28 \\
336 & 140 & 2 & 28 \\
140 & 336 & 2 & 28 \\
140 & 336 & 2 & 28 \\
28 & 140 & 2 & 28 \\
28 & 56 & 2 & 28 \\
56 & 140 & 2 & 28 \\
140 & 56 & 2 & 28 \\
56 & 28 & 2 & 28 \\
28 & 0 & 2 & 28 \\
\end{array}
\]

\( GCF(2389, 485, 59675) = GCF(59675, 2485) = GCF(2485, 35) = GCF(35, 0) = 35 \)

6) a) \( 32 = 2^5 \)
   \( 1024 = 2^{10} \)
   So \( LCM(32, 1024) = 2^{10} = 1024 \)

(This makes sense since 1024 is a multiple of 32; namely 1024 = 32 \cdot 32)

b) \( 24 = 2^3 \cdot 3 \)
   \( 120 = 2^3 \cdot 3 \cdot 5 \)
   \( 1056 = 2^5 \cdot 3 \cdot 11 \)
   So \( LCM(24, 120, 1056) = 2^5 \cdot 3 \cdot 5 \cdot 11 \)
   = 5280
7) a) (5A, p. 38-39) Common multiples are being used as "common denominators" when adding/subtracting fractions.

b) \[ 84 = 2^2 \cdot 3 \cdot 7 \quad \text{and} \quad 147 = 3 \cdot 7^2 \]

So \[ \text{LCM}(84, 147) = 2^2 \cdot 3 \cdot 7^2 = 588 \]

Now \( \frac{2}{84} + \frac{5}{147} \) = \( \frac{14}{588} + \frac{20}{588} = \frac{34}{588} \) (which reduces to \( \frac{17}{294} \))

- Mutl. top/bottom by 7 to write as a fraction with denom 588
- Mutl. top/bottom by 4 to write as a fraction with denom 588

9) Find LCM (192, 320)

\[ 192 = 2^5 \cdot 3 \quad \text{and} \quad 320 = 2^6 \cdot 5 \]

So \[ \text{LCM}(192, 320) = 2^6 \cdot 3 \cdot 5 = 960 \]

We need 5 revolutions of the first gear to realign the mark.

\[ 960 \div 192 \]

(and 3 revolutions of the second gear)

\[ 960 \div 3 \]

10) We must find a number \(< 1000\) that leaves remainder 1 upon division by 2, 3, 4, 5, 6, 7, or 8.

HINT: The number must follow one that is divisible by 2, 3, 4, 5, 6, 7, 8; that is, to find the number, ADD 1 to LCM (2, 3, 4, 5, 6, 7, 8).

\[
\begin{align*}
2 &= 2^1 \\
3 &= 3^1 \\
4 &= 2^2 \\
5 &= 5^1 \\
6 &= 2^1 \cdot 3^1 \\
7 &= 7^1 \\
8 &= 2^3
\end{align*}
\]

So \[ \text{LCM} = 2^3 \cdot 3^1 \cdot 5^1 \cdot 7^1 = 840 \]

* The number we need is 840 + 1 or \( \boxed{841} \)

\[ 840 \text{ is the smallest multiple these numbers have in common; } 840 \text{ is the smallest number divisible by every number given.} \]