1) a) Find a common multiple of 15 and 12

The multiples of 15 are 15, 30, 45, 60, ...
60 is also a multiple of 12 So 60 is a common multiple of 15 and 12

b) (4A, Practice 1B, p 27)

Problem 1 FACTORS of 18: 1, 2, 3, 6, 9, 18

Problem 4 a) factors of 8: 1, 2, 4, 8
b) factors of 15: 1, 3, 5, 15
c) factors of 20: 1, 2, 4, 5, 10, 20
d) factors of 50: 1, 2, 5, 10, 25, 50
e) factors of 75: 1, 3, 5, 15, 25, 75
f) factors of 98: 1, 2, 7, 14, 49, 98

Problem 5 (find a common factor different from 1)
a) 3 is a common factor of 15 and 6
b) 2 and 4 are common factors of 12 and 16
c) 3 is a common factor of 15 and 18

Problem 6 a) multiples of 2: 2, 4, 6, 8, ...
b) multiples of 6: 6, 12, 18, 24, ...
c) multiples of 8: 8, 16, 24, 32, ...

Problem 7 a) The multiples of 4 are 4, 8, 12, ...
12 is also a multiple of 3 So 12 is a common multiple of 3 and 4

b) The multiples of 5 are 5, 10, 15, 20, ...
20 is also a multiple of 4 So 20 is a common multiple of 4 and 5

c) 12 is a common multiple of 4 and 6

2) PROVE that if b is a multiple of a (that is, $a|b$), then $GCF(a,b) = a$

FIRST THINK OF A SPECIFIC EXAMPLE WITH ONE NUMBER A MULTIPLE OF THE OTHER TO SEE WHAT THE STATEMENT IS SAYING; for ex $GCF(3,27) = 3$ (and $LCM(3,27) = 27$)

a is the greatest number that divides a. Since we are given a divides b, a must be the greatest divisor a and b have in common; that is, $GCF(a,b) = a$

(alternatively: Since $GCF(a,b)$ must divide a, we must have $GCF(a,b) \le a$ We are given $a|b$ Since $a|a$, a is a common factor of a and b Since $GCF(a,b)$ is the greatest common factor of a and b, $GCF(a,b) \ge a$. Putting these together, $GCF(a,b) = a$)

(Note If $a|b$, we can also conclude $LCM(a,b) = b$
b is the smallest (pos) multiple of b Since we are given $a|b$, b is a multiple of a. So b must be the smallest multiple a and b have in common; that is, $LCM(a,b) = b$)

3) Prove that if p is prime, then $GCF(p,a) = 1$ unless a is a multiple of p

The only divisors of a prime p are 1 and p

If $p \mid a$ (that is, a is a multiple of p), then p is the greatest divisor p and a have in common. So $GCF(p,a) = p$ in this case.

If $p \nmid a$, then the only divisor p and a have in common is 1

So $GCF(p,a) = 1$ in this case.

4) a) $28 = 2^2 \cdot 7$
$63 = 3^2 \cdot 7$
So $GCF(28,63) = 7$

b) $104 = 2^3 \cdot 13$
$132 = 2^2 \cdot 3 \cdot 11$
So $GCF(104,132) = 2^2 = 4$

c) $24 = 2^3 \cdot 3$
$56 = 2^3 \cdot 7$
$180 = 2^2 \cdot 3^2 \cdot 5$
$GCF(24,56,180) = 2^2 = 4$

5) EUCLID'S ALGORITHM

a) $GCF(91,52) = GCF(52,39) = GCF(39,13) = GCF(13,0) = 13$

$$52\overline{)91}^{\,1}\quad 91-52=39 \qquad 39\overline{)52}^{\,1}\quad 52-39=13 \qquad 13\overline{)39}^{\,3}\quad 39-39=0$$

LAST NONZERO REMAINDER turns out to be the GCF

b) $GCF(812,336) = GCF(336,140) = GCF(140,56) = GCF(56,28)$
$= GCF(28,0) = 28$

$$336\overline{)812}^{\,2}\quad 812-672=140 \qquad 140\overline{)336}^{\,2}\quad 336-280=56 \qquad 56\overline{)140}^{\,2}\quad 140-112=28 \qquad 28\overline{)56}^{\,2}\quad 56-56=0$$

c) $GCF(2389485, 59675) = GCF(59675, 2485) = GCF(2485, 35)$
$= GCF(35, 0) = 35$

6) a) $\left.\begin{aligned} 32 &= 2^5 \\ 1024 &= 2^{10} \end{aligned}\right\}$ So $LCM(32,1024) = 2^{10} = 1024$
(This makes sense since 1024 is a multiple of 32; namely $1024 = 32 \cdot 32$

b) $\left.\begin{aligned} 24 &= 2^3 \cdot 3 \\ 120 &= 2^3 \cdot 3 \cdot 5 \\ 1056 &= 2^5 \cdot 3 \cdot 11 \end{aligned}\right\}$ So $LCM(24,120,1056) = 2^5 \cdot 3 \cdot 5 \cdot 11$
$= 5280$

7) a) (5A, p38-39) Common multiples are being used as "common denominators" when adding / subtracting fractions.

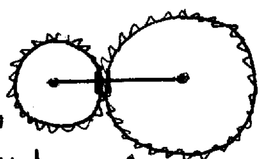
b) $\left.\begin{array}{l} 84 = 2^2 \cdot 3 \cdot 7 \\ 147 = 3 \cdot 7^2 \end{array}\right\}$ so $\text{LCM}(84, 147) = 2^2 \cdot 3 \cdot 7^2 = 588$

Now $\frac{2}{84} + \frac{5}{147} = \frac{14}{588} + \frac{20}{588} = \frac{34}{588}$ (which reduces to $\frac{17}{294}$)

mult top/bottom by 7 to write as a fraction with denom 588

mult top/bottom by 4 to write as a fraction with denom 588

9)



Gear 1
192 teeth

Gear 2
320 teeth

Find LCM (192, 320)

$\left.\begin{array}{l} 192 = 2^5 \cdot 3 \\ 320 = 2^6 \cdot 5 \end{array}\right\}$ so $\text{LCM}(192, 320) = 2^6 \cdot 3 \cdot 5 = 960$

We need 5 revolutions of the first gear to realign the mark

$960 \div 192$

(and 3 revolutions of the second gear)

$960 \div 3$

10) We must find a number < 1000 that leaves remainder 1 upon division by 2, 3, 4, 5, 6, 7 or 8

HINT: The number must follow one that is divisible by 2, 3, 4, 5, 6, 7, 8; that is, to find the number, ADD 1 to LCM (2, 3, 4, 5, 6, 7, 8)

$\left.\begin{array}{l} 2 = 2^1 \\ 3 = 3^1 \\ 4 = 2^2 \\ 5 = 5^1 \\ 6 = 2^1 \cdot 3^1 \\ 7 = 7^1 \\ 8 = 2^3 \end{array}\right\}$ So $\text{LCM} = 2^3 \cdot 3^1 \cdot 5^1 \cdot 7^1 = 840$

* The number we need is $840 + 1$ or 841

[840 is the smallest multiple these numbers have in common; 840 is the smallest number divisible by EVERY number given]