1) Test if the following numbers are PRIME. If not prime, provide a factorization

a) 127 check divisibility by 2, 3, 5, 7, 11 ($13^2 = 169 > 127$)
127 is not divisible by any prime on our list → 127 is PRIME

b) 129 same list as in a) check divisibility by 2, 3, 5, 7, 11 ($13^2 = 169 > 129$)
$2 \nmid 129$, $3 \mid 129$ STOP 129 is COMPOSITE $129 = 3 \cdot 43$

c) 327 check divisibility by 2, 3, 5, 7, 11, 13, 17 ($19^2 = 361 > 327$)
(NOTE $\sqrt{327} \sim 18.1$, so 17 is the largest prime we'll need to consider)
$2 \nmid 327$, $3 \mid 327$ STOP 327 is COMPOSITE $327 = 3 \cdot 109$

d) 221 check divisibility by 2, 3, 5, 7, 11, 13 ($17^2 = 289 > 221$)
(NOTE $\sqrt{221} \sim 14.9$ or you can estimate $\sqrt{221} \sim \sqrt{225} = 15$)
$2 \nmid 221$, $3 \nmid 221$, $5 \nmid 221$, $7 \nmid 221$, $11 \nmid 221$, $13 \mid 221$ 221 is COMPOSITE
$221 = 13 \cdot 17$

e) 337 check divisibility by 2, 3, 5, 7, 11, 13, 17 ($19^2 = 361 > 337$)
(NOTE $\sqrt{337} \sim 18.4$)
337 is not divisible by any prime on our list → 337 is PRIME

f) 389 ($\sqrt{389} \sim 19.7$)
check divisibility by 2, 3, 5, 7, 11, 13, 17, 19 ($23^2 = 529 > 389$)
389 is not divisible by any prime on our list → 389 is PRIME

g) 223 check divisibility by 2, 3, 5, 7, 11, 13 ($17^2 = 289 > 223$)
(NOTE $\sqrt{223} \sim 14.9$ or $\sqrt{223} \sim \sqrt{225} = 15$)
223 is not divisible by any prime on our list → 223 is PRIME

h) 859 ($\sqrt{859} \sim 29.3$) check divisibility by 2, 3, 5, 7, 11, 13, 17, 19, 23, 29
859 is not divisible by any prime on our list → 859 is PRIME

2) Find the largest prime less than 1000.
999 is COMPOSITE (since $3 \mid 999$)
998 is COMPOSITE (since $2 \mid 998$) We can skip EVEN numbers (any even number >2 is composite)
997 is PRIME! ($\sqrt{997} \sim 31.6$) Check divisibility by 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31
997 is not divisible by any prime on our list

3) a) 9 is <u>odd</u> but 9 is <u>not</u> prime; 9 is composite since $3|9$
$9 = 3 \cdot 3$

b) For example $\underbrace{(2+3)^2}_{\substack{\text{this equals} \\ 5^2 \\ \text{or } 25}} \neq \underbrace{2^2+3^2}_{\substack{\text{this equals} \\ 4+9 \\ \text{or } 13}}$

c) Tiana isn't right - she's basing her conjecture on a few values of n
If she tried $n=10$, then $n^2+n+11 = 10^2+10+11 = 121$ which is <u>composite</u> $(121 = 11 \cdot 11)$
Notice if she tried $n=11$, then $n^2+n+11 = \underbrace{11^2+11+11}_{\substack{\text{you can see this is a} \\ \text{multiple of 11 just by} \\ \text{factoring out 11} \\ 11(11+1+1) \text{ or } 11 \cdot 13}} = 143$ which is <u>composite</u> $(143 = 11 \cdot 13)$

4) Prove that the <u>sum</u> of any three consecutive numbers is divisible by 3
If x is the first number, the next two numbers are $x+1$ and $x+2$
(so x, $x+1$, $x+2$ are our three consecutive numbers)
$\underbrace{x + (x+1) + (x+2)}_{\text{the SUM of our numbers.}} = 3x+3 = \underline{3}(x+1)$, which shows the sum is a multiple of 3; that is, 3 divides the sum

5) NOTICE $\boxed{6! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6}$ is divisible by 2, 3, 4, 5, 6
★ CORRECT MISPRINT IN TEXT

a) $6! + 2 = (1 \cdot \underline{2} \cdot 3 \cdot 4 \cdot 5 \cdot 6) + \underline{2} = \underline{2} \cdot ((1 \cdot 3 \cdot 4 \cdot 5 \cdot 6) + 1)$
So 2 is a factor of $6!+2$

$6! + 3 = (1 \cdot 2 \cdot \underline{3} \cdot 4 \cdot 5 \cdot 6) + \underline{3} = \underline{3} \cdot ((1 \cdot 2 \cdot 4 \cdot 5 \cdot 6) + 1)$
So 3 is a factor of $6!+3$

$6! + 4 = (1 \cdot 2 \cdot 3 \cdot \underline{4} \cdot 5 \cdot 6) + \underline{4} = \underline{4} \cdot ((1 \cdot 2 \cdot 3 \cdot 5 \cdot 6) + 1)$
So 4 is a factor of $6!+4$

$6! + 5 = (1 \cdot 2 \cdot 3 \cdot 4 \cdot \underline{5} \cdot 6) + \underline{5} = \underline{5} \cdot ((1 \cdot 2 \cdot 3 \cdot 4 \cdot 6) + 1)$
So 5 is a factor of $6!+5$

$6! + 6 = (1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot \underline{6}) + \underline{6} = \underline{6} \cdot ((1 \cdot 2 \cdot 3 \cdot 4 \cdot 5) + 1)$
So 6 is a factor of $6!+6$

cont →

Notice $6!+2,\ 6!+3,\ 6!+4,\ 6!+5,\ 6!+6$

is a list of 5 consecutive COMPOSITE numbers

has a factor of 2 — has a factor of 3 — has a factor of 4 — has a factor of 5 — has a factor of 6

b) Name a factor of $31!+29$

29 is a factor since $31!+29 = \underline{29}\cdot\left((1\cdot 2\cdot 3\cdots 28\cdot 30\cdot 31)+1\right)$

c) Find 5000 consecutive COMPOSITE numbers

* MISPRINT in HINT: start with $5001!+2$

$5001!+2,\ 5001!+3,\ 5001!+4,\ \ldots\ldots,\ 5001!+5000,\ 5001!+5001$

This is a list of 5000 consecutive composite numbers

How many numbers are in our list?

Since we start at $5001!+\underline{2}$ and go to $5001!+\underline{5001}$

we have 5000 numbers (if we stopped at $5001!+5000$, only have 4999)

Why are they <u>COMPOSITE</u>?

2 is a factor of $5001!+2$

3 is a factor of $5001!+3$

$\vdots$

5000 is a factor of $5001!+5000$

5001 is a factor of $5001!+5001$

<u>Note</u> - if we did as suggested

$5000!+2,\ 5000!+3,\ \ldots,\ 5000!+5000$

we'd only have a list of 4999 consecutive composite numbers

<u>In general</u> to create a list of n consecutive composite numbers, you can use

$(n+1)!+2,\ (n+1)!+3,\ \ldots,\ (n+1)!+n,\ (n+1)!+(n+1)$