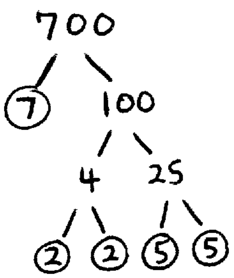
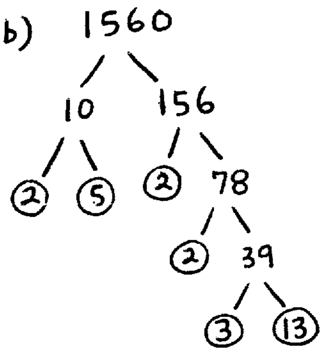
HW Set 21 — Section 5.3 — p 121

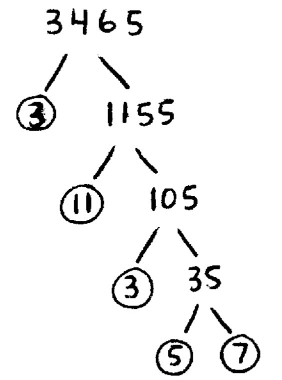
1) Find the PRIME FACTORIZATION

a)


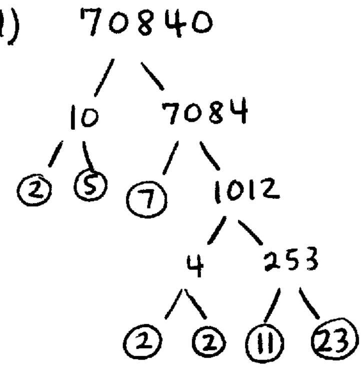
$700 = 2^2 \cdot 5^2 \cdot 7$

b)


$1560 = 2^3 \cdot 3 \cdot 5 \cdot 13$

c)


$3465 = 3^2 \cdot 5 \cdot 7 \cdot 11$

d)


$70840 = 2^3 \cdot 5 \cdot 7 \cdot 11 \cdot 23$

2) Determine if prime (P), composite (C), or neither

a) 12 C 2 is a factor of 12 that's different from 1, different from 12

b) 123 C $3 \mid 123$ (sum of digits = 6 and $3 \mid 6$)

c) 1234 C $2 \mid 1234$ (note: any even number > 2 is COMPOSITE)

d) 12345 C $5 \mid 12345$

e) 154 C $2 \mid 154$

f) 102302320 C 2 is a factor

g) 1 NEITHER (A prime has exactly two (pos) divisors; a composite has more than two (pos) divisors. 1 has exactly one (pos) divisor, namely 1)

h) 97 P The only divisors of 97 are 1 and 97 (We'll see in 5.4 (TEST FOR PRIMENESS) that it suffices to check if 97 is divisible by 2, 3, 5, 7. Since none of these divide 97, we can conclude 97 is prime)

3)
a) $63 = 3^2 \cdot 7$

b) $768 = 2^8 \cdot 3$

c) $324 = 2^2 \cdot 3^4$

d) $361 = 19^2$

e) $196 = 2^2 \cdot 7^2$

f) $1024 = 2^{10}$

g) $480 = 2^5 \cdot 3 \cdot 5$

h) $10{,}000 = 2^4 \cdot 5^4$

$10{,}000 = 10^4 = (2 \cdot 5)^4 = 2^4 \cdot 5^4$

5) a) $10! = 1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\cdot9\cdot10$

10 factorial $= 1\cdot2\cdot3\cdot(2\cdot2)\cdot5\cdot(2\cdot3)\cdot7\cdot(2\cdot2\cdot2)\cdot(3\cdot3)\cdot(2\cdot5)$

$= 2^8\cdot3^4\cdot5^2\cdot7$

b) Is 10! divisible

i) by 10? YES since $10 = 2\cdot5$

ii) by 30? YES since $30 = 2\cdot3\cdot5$

iii) by 120? YES since $120 = 2^3\cdot3\cdot5$

iv) by 1000? <u>NO</u> since $1000 = 2^3\cdot5^{3}$

here's why
The power on 5 in 1000 is <u>larger</u> than the power on 5 in 10!

* KEY to b) Any divisor of 10! must look like $2^r\cdot3^s\cdot5^t\cdot7^w$ where $0\le r\le 8$ (r can be any whole no. from 0 to 8) $0\le s\le 4$ (s can be any whole no from 0 to 4) $0\le t\le 2$ (t can be 0, 1, or 2) $0\le w\le 1$ (w can be 0 or 1)

Alternatively

10! is divisible by 10 since $10! = \underline{10}\cdot(1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\cdot9)$

10! is divisible by 30 since $10! = (\underbrace{3\cdot10}_{=\,30})(1\cdot2\cdot4\cdot5\cdot6\cdot7\cdot8\cdot9)$

10! is divisible by 120 since $10! = (\underbrace{3\cdot5\cdot8}_{=\,120})(1\cdot2\cdot4\cdot6\cdot7\cdot9\cdot10)$

c) Is 30! divisible by 2,400,000? YES

Think of 30 factorial

Can we write 30! as a multiple of $24\cdot10^5$ or $24\cdot2^5\cdot5^5$

$30! = 1\cdot2\cdot3\cdot4\cdot\boxed{5}\cdots8\cdot9\cdot\underbrace{10}_{=\,2\cdot\boxed{5}}\cdots\underbrace{15}_{=\,3\cdot\boxed{5}}\cdots24\cdot\underbrace{25}_{=\,\boxed{5}\cdot\boxed{5}}\cdots30$

here's 4·8 which is 2^5 or 32

here are five 5's or 5^5

here's 24

So $30! = 24\cdot2^5\cdot5^5\cdot$(remaining factors) $= 2,400,000\cdot$(remaining factors)

continued →

d) Find the largest N such that $18!$ is divisible by 12^N

$$18! = \underbrace{1\cdot 2\cdot 3\cdot \ldots\ldots 10}_{\text{in a) we saw that } 10! = 2^8 3^4 5^2 7}\cdot 11\cdot 12\cdot 13\cdot 14\cdot 15\cdot 16\cdot 17\cdot 18$$

So $18! = (2^8 3^4 5^2 7)\cdot 11\cdot (2^2\cdot 3)\cdot 13\cdot (2\cdot 7)\cdot (3\cdot 5)\cdot (2^4)\cdot 17\cdot (2\cdot 3^2)$

So $18! = 2^{16} 3^8\cdot 5^3\cdot 7^2\cdot 11\cdot 13\cdot 17$

Now $12^N = (2^2\cdot 3)^N = 2^{2N}\cdot 3^N$

Based on the powers on 2 and 3 in the prime factorization of $18!$, the largest N for which $18!$ is divisible by 12^N is $\boxed{N=8}$

$12^8 = 2^{2(8)}\cdot 3^8 = 2^{16}\cdot 3^8$, so 12^8 is a factor of $18!$

$12^9 = 2^{18}\cdot 3^9$, so 12^9 is not a factor of $18!$

These powers are too big! They're both larger than the respective powers on 2 and 3 in $18!$