To simplify notation, write $k \mid A$ for the statement "k divides A"

1) Test for divisibility by 3? by 9? by 11?

a) 2,838 sum of digits = 21 $3 \mid 21$, so DIV BY 3
$9 \nmid 21$, so NOT DIV BY 9

by 11? sum of digits in even powers of 10 spots = $8+8=16$
sum of digits in odd powers of 10 spots = $2+3=5$
Does $11 \mid (16-5)$? Yes $11 \mid 11$, so DIV BY 11

b) 34,521 sum of digits = 15 $3 \mid 15$, so DIV BY 3
$9 \nmid 15$, so NOT DIV BY 9

by 11? sum of digits in even powers of 10 spots = $3+5+1=9$
sum of digits in odd powers of 10 spots = $4+2=6$
Does $11 \mid (9-6)$? No $11 \nmid 3$, so NOT DIV BY 11

c) 10,234,341 sum of digits = 18 $3 \mid 18$, so DIV BY 3
$9 \mid 18$, so DIV BY 9 (once you know DIV BY 9, it must be DIV BY 3)

by 11? $0+3+3+1=7$ $11 \nmid (11-7)$, so NOT DIV BY 11
$1+2+4+4=11$

d) 792 DIV BY 3, DIV BY 9, DIV BY 11 (since $11 \mid ((7+2)-9)$ or $11 \mid 0$)

e) 8,394 DIV BY 3, NOT DIV BY 9, NOT DIV BY 11

f) 26,341 NOT DIV BY 3, NOT DIV BY 9, NOT DIV BY 11

g) 333,333 DIV BY 3, DIV BY 9, DIV BY 11 (since $11 \mid (\underset{3+3+3}{9} - \underset{3+3+3}{9})$ or $11 \mid 0$)

h) 179 NOT DIV BY 3, NOT DIV BY 9, NOT DIV BY 11

2) Test 5,192,132 for divisibility by 3, 4, 5, 8, 9, 11

last digit is a 2 (not a 0 or 5), so NOT DIV BY 5

sum of digits = 23 $3 \nmid 23$, so NOT DIV BY 3
$9 \nmid 23$, so NOT DIV BY 9 (once you know NOT DIV BY 3, it can't be divisible by 9)

$4 \mid 32$ ← number represented by LAST TWO DIGITS so DIV BY 4

$8 \nmid 132$ ← number represented by LAST THREE DIGITS so NOT DIV BY 8

sum of digits in even powers of 10 spots = $5+9+1+2=17$
sum of digits in odd powers of 10 spots = $1+2+3=6$
Does $11 \mid (17-6)$? Yes $11 \mid 11$, so DIV BY 11

3) Test 186,426 for divisibility by 2, 3, 4, 5, 8, 9, 10, 11

last digit is a 6 $\rightarrow$ DIV BY 2, NOT DIV BY 5, NOT DIV BY 10

sum of digits = 27 $\quad 3 \mid 27 \rightarrow$ DIV BY 3 $\quad$ (once you know DIV BY 9, it must be DIV BY 3)

$\quad 9 \mid 27 \rightarrow$ DIV BY 9

$4 \nmid 26 \rightarrow$ NOT DIV BY 4 $\quad$ (once you know NOT DIV BY 4, it <u>can't</u> be divisible by 8)

$8 \nmid 426 \rightarrow$ NOT DIV BY 8

Does $11 \mid (\underbrace{18}_{8+4+6} - \underbrace{9}_{1+6+2})$? No $11 \nmid 9$, so NOT DIV BY 11

4) (4A, p25) Divisibility Tests for 2, for 5, for 3

5) It is true that if $18 \mid m$, then $3 \mid m$ and $6 \mid m$

(<u>reason</u>: If $18 \mid m$, then $m = 18 \cdot n$ for some whole number n
So $m = 3 \cdot 6 \cdot n$ which shows $3 \mid m$ and $6 \mid m$)

The converse is not necessarily true; for example,

$3 \mid 24$ and $6 \mid 24$ but $18 \nmid 24$

(that is, 24 is divisible by both 3 and 6, but is <u>not</u> divisible by 18)

(<u>Note</u> - there is a divisibility test for 18
A number is divisible by 18 if and only if it is divisible by both 2 and 9
The reason we can use 2 and 9 is that the only common factor of 2 and 9 is 1 (this isn't true for the pair 3 and 6; 3 and 6 have a common factor different from 1, namely 3))