To simplify notation, write \( A | A \) for the statement "\( k \) divides \( A \)"

1) Test for divisibility by 3? by 9? by 11?

a) 2,838 sum of digits = 21 \( 3 \) divides 21, so DIV BY 3
   \( 9 \) does not divide 21, so NOT DIV BY 9
   by 11? sum of digits in even powers of 10 spots = \( 8 + 8 = 16 \)
   sum of digits in odd powers of 10 spots = \( 2 + 3 = 5 \)
   Does 11 \( \mid (16 - 5) \) ? Yes 11 \( \mid 11 \), so DIV BY 11

b) 34,521 sum of digits = 15 \( 3 \) divides 15, so DIV BY 3
   \( 9 \) does not divide 15, so NOT DIV BY 9
   by 11? sum of digits in even powers of 10 spots = \( 3 + 5 + 1 = 9 \)
   sum of digits in odd powers of 10 spots = \( 4 + 2 = 6 \)
   Does 11 \( \mid (9 - 6) \) ? No 11 \( \not\mid 3 \), so NOT DIV BY 11

c) 10,234,341 sum of digits = 18 \( 3 \) divides 18, so DIV BY 3
   \( 9 \) divides 18, so DIV BY 9
   by 11? \( 0 + 3 + 3 + 4 = 7 \) \( 11 \not\mid (7 - 3) \), so NOT DIV BY 11
   \( 1 + 2 + 4 + 1 = 11 \)

d) 792 DIV BY 3, DIV BY 9, DIV BY 11 (since 11 \( \mid (7+2-9) \) or 11 \( \mid 0 \))

e) 8,394 DIV BY 3, NOT DIV BY 9, NOT DIV BY 11

f) 26,341 NOT DIV BY 3, NOT DIV BY 9, NOT DIV BY 11

g) 333,333 DIV BY 3, DIV BY 9, DIV BY 11 (since 11 \( \mid (9-9) \) or 11 \( \mid 0 \))

h) 179 NOT DIV BY 3, NOT DIV BY 9, NOT DIV BY 11

2) Test 5,192,132 for divisibility by 3, 4, 5, 8, 9, 11

last digit is a 2 (not a 0 or 5), so NOT DIV BY 5

sum of digits = 23 \( 3 \not\mid 23 \), so NOT DIV BY 3
   \( 9 \) does not divide 23, so NOT DIV BY 9
   (once you know NOT DIV BY 3, it can't be divisible by 9)

4 \( \mid 32 \) \rightarrow number represented by LAST TWO DIGITS

8 \( \mid 132 \) \rightarrow number represented by LAST THREE DIGITS

sum of digits in even powers of 10 spots = \( 5 + 9 + 1 + 2 = 17 \)
sum of digits in odd powers of 10 spots = \( 1 + 2 + 3 = 6 \)
Does 11 \( \mid (17 - 6) \) ? Yes 11 \( \mid 11 \), so DIV BY 11
3) Test 186,426 for divisibility by 2, 3, 4, 5, 8, 9, 10, 11
   last digit is a 6 → DIV BY 2, NOT DIV BY 5, NOT DIV BY 10
   sum of digits = 27  3|27 → DIV BY 3 (once you know DIV BY 9,)
                       9|27 → DIV BY 9 (it must be DIV BY 3)
   4∤26 → NOT DIV BY 4 (once you know NOT DIV BY 4)
   8∤426 → NOT DIV BY 8 (it can't be divisible by 8)

   Does 11 | (18 - 9)?  No 11∤9, so NOT DIV BY 11
   \[ 8+4+6 \]

4) (4A, p25) Divisibility Tests for 2, for 5, for 3

5) It is true that if 18|m, then 3|m and 6|m
   (reason: If 18|m, then m = 18 \cdot n for some whole number n)
   \[ So m = 3 \cdot 6 \cdot n \] which shows 3|m and 6|m

   The converse is not necessarily true; for example,
   3|24 and 6|24 but 18∤24
   (that is, 24 is divisible by both 3 and 6, but is not divisible by 18)

   Note - there is a divisibility test for 18
   A number is divisible by 18 if and only if it is divisible by
   both 2 and 9
   The reason we can use 2 and 9 is that the only common factor
   of 2 and 9 is 1 (this isn't true for the pair 3 and 6; 3 and 6
   have a common factor different from 1, namely 3)