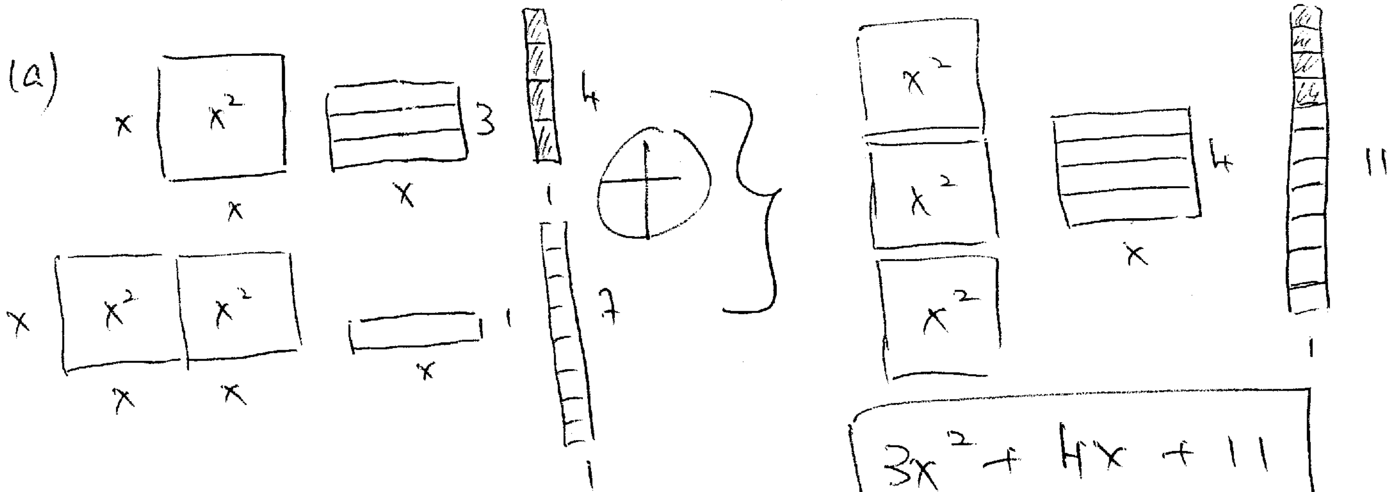
3) $(x^2 + 3x + 4) + (2x^2 + x + 7)$

(a)



$\boxed{3x^2 + 4x + 11}$

(b) $(x^2 + 3x + 4) + (2x^2 + x + 7)$

$= x^2 + 2x^2 + 3x + x + 4 + 7$ any-order property (really COMM and ASSOC Props)

$= (1+2)x^2 + (3+1)x + (4+7)$ Distributive property

$= 3x^2 + 4x + 11$ addition.

4) Using $(a+b)^2 = a^2 + 2ab + b^2$

$21^2 = (20+1)^2 = 20^2 + 2\cdot 20\cdot 1 + 1^2 = 400 + 40 + 1 = 441$

$31^2 = (30+1)^2 = 30^2 + 2\cdot 30\cdot 1 + 1^2 = 900 + 60 + 1 = 961$

$41^2 = (40+1)^2 = 1600 + 80 + 1 = 1681$

$51^2 = (50+1)^2 = 2500 + 100 + 1 = 2601$

Using $(a-b)^2 = a^2 - 2ab + b^2$

$19^2 = (20-1)^2 = 20^2 - 2\cdot 20\cdot 1 + 1^2 = 400 - 40 + 1 = 361$

$29^2 = (30-1)^2 = 30^2 - 2\cdot 30\cdot 1 + 1^2 = 900 - 60 + 1 = 841$

$39^2 = (40-1)^2 = 1600 - 80 + 1 = 1521$

$49^2 = (50-1)^2 = 2500 - 100 + 1 = 2401$

5) $15^2 = 225$

$150^2 = 22500$

$151^2 = (150+1)^2$

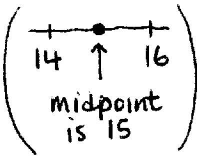
$= 150^2 + 2\cdot 150\cdot 1 + 1^2$

$= 22500 + 300 + 1 = 22,801$

6) ALL involve $(a+b)(a-b) = a^2 - b^2$

a) <u>numbers which differ by 2</u> $(a+1)(a-1) = a^2 - 1$

$16 \times 14 = (15+1)(15-1) = 15^2 - 1^2 = 225 - 1 = 224$



$20 \times 18 = (19+1)(19-1) = 19^2 - 1^2 = 361 - 1 = 360$

$15 \times 13 = (14+1)(14-1) = 14^2 - 1^2 = 196 - 1 = 195$

$19 \times 17 = (18+1)(18-1) = 18^2 - 1^2 = 324 - 1 = 323$

$18 \times 16 = (17+1)(17-1) = 17^2 - 1^2 = 289 - 1 = 288$

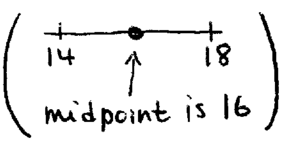
$21 \times 19 = \quad = 20^2 - 1 = 400 - 1 = 399$

$141 \times 139 = \quad = 140^2 - 1 = 19600 - 1 = 19,599$

$8 \times 6 = \quad = 7^2 - 1 = 49 - 1 = 48$

b) <u>numbers which differ by 4</u> $(a+2)(a-2) = a^2 - 4$

$18 \times 14 = (16+2)(16-2) = 16^2 - 2^2 = 256 - 4 = 252$



$16 \times 12 = (14+2)(14-2) = 14^2 - 2^2 = 196 - 4 = 192$

$17 \times 13 = (15+2)(15-2) = 15^2 - 2^2 = 225 - 4 = 221$

$21 \times 17 = (19+2)(19-2) = 19^2 - 2^2 = 361 - 4 = 357$

$13 \times 9 = \quad = 11^2 - 2^2 = 121 - 4 = 117$

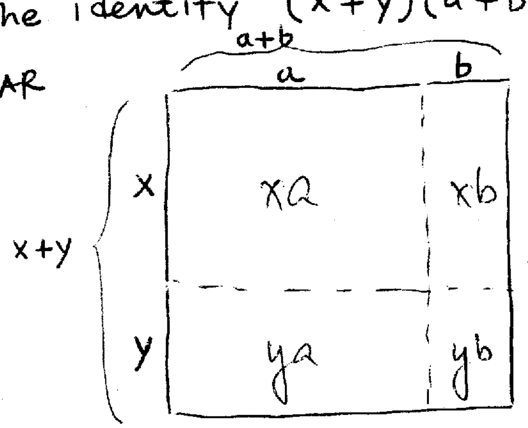
$62 \times 58 = \quad = 60^2 - 2^2 = 3600 - 4 = 3596$

$192 \times 188 = \quad = 190^2 - 2^2 = 36100 - 4 = 36,096$

$8 \times 4 = \quad = 6^2 - 2^2 = 36 - 4 = 32$

7) Consider the identity $(x+y)(a+b) = xa + xb + ya + yb$

a) RECTANGULAR ARRAY



AREA of rectangle with sides $x+y$ and $a+b$

$= (x+y)(a+b)$

$=$ sum of areas of four (rect.) pieces

$= xa + xb + ya + yb$

continued →

b) Using DISTRIBUTIVE PROPERTY

$$(x+y)(a+b) = (x+y)a + (x+y)b$$
$$= xa + ya + xb + yb$$
$$= xa + xb + ya + yb$$

using comm of +
$ya + xb = xb + ya$

Alternatively

$$(x+y)(a+b) = x(a+b) + y(a+b)$$
$$= xa + xb + ya + yb$$

(but note that it doesn't work in general; better to think of DISTRIB PROP for ex $(x+y+z)(a+b)$)

c) Does the FOIL mnemonic work here? YES

$$(x+y)(a+b)$$

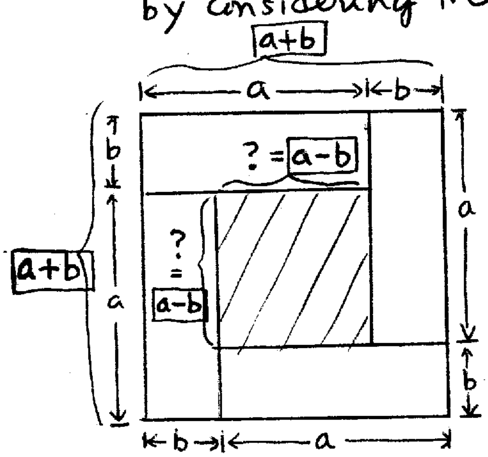
'F'	First	xa
'O'	Outer	xb
'I'	Inner	ya
'L'	Last	yb

8) By repeatedly using DISTRIB Prop, derive an identity for $(a+b)^3$

$$(a+b)^3 = (a+b)\big((a+b)(a+b)\big)$$
$$= (a+b)\big((a+b)a + (a+b)b\big)$$
$$= (a+b)(a^2 + ba + ab + b^2)$$
$$= (a+b)(a^2 + 2ab + b^2) \quad \text{(using comm prop of mult } ba = ab\text{)}$$
$$= a(a^2 + 2ab + b^2) + b(a^2 + 2ab + b^2)$$
$$= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3$$
$$= \boxed{a^3 + 3a^2b + 3ab^2 + b^3}$$

Note
We've done $(a+b)^2$
$(a+b)^3$
These are special cases of the BINOMIAL THEOREM which gives $(a+b)^n$

9) Write an identity for $(a+b)^2 - (a-b)^2$ by considering the figure



The KEY is to first figure out the side of the SHADED IN SQUARE INSIDE $\quad ? = \boxed{a-b}$

Next
$$(a+b)^2 - (a-b)^2$$
= AREA of Large Square — AREA of small square inside
= sum of the areas of the four rectangles (making the border)
$$= ab + ab + ab + ab$$
$$= 4ab \qquad \text{So } \boxed{(a+b)^2 - (a-b)^2 = 4ab}$$

Alternatively
Using identities we're already done (used in Problem 4)

$$(a+b)^2 - (a-b)^2 = (a^2 + 2ab + b^2) - (a^2 - 2ab + b^2)$$
$$= \cancel{a^2} + 2ab + \cancel{b^2} - \cancel{a^2} + 2ab - \cancel{b^2} = 4ab$$