Math 592D Extra Assignment

Time Dilation for a Traveler Accelerated by One Earth Gravity

Sam is in outer space near the earth in a space ship with engines that can give it constant acceleration or deceleration of 1 earth gravity (relative to its own instantaneous inertial frame). Because of this Sam can always walk around comfortably in his space ship during his trips with his usual weight just as he would on earth.

Sam is planning a trip to the center of the Milky Way Galaxy (our galaxy) 30,000 light years away. For the trip he will accelerate with constant acceleration 1g for the first half of the trip and then decelerate at 1 g for the second half of the trip coming to rest near the center of the galaxy. He will then return to earth in the same way, accelerating at 1 g for half the return trip and then decelerating at 1 g for the final leg of the journey.

The goal of this problem is to determine how much older Sam is at the end of this trip and what calendar year he arrives back on earth. Follow the steps below to find the answer. Assume throughout that the earth's frame is inertial (this isn't quite true, but it's good enough for this problem).

1. The acceleration of gravity at the surface of the earth is \( g = 9.8 \text{ m/sec}^2 \). Verify that (by lucky numerical coincidence), \( g \approx 1 \text{ light year/year}^2 \). Use this value for \( g \) (actually this is slightly less than one earth gravity, but close enough for comfort on a spaceship). For this problem, all time will be measured in years.

2. It is sufficient to consider only 2 spacetime dimensions for this problem, time and Sam's linear distance from earth. Using part 1, write a formula for Sam's worldcurve \( X(\tau) \), as measured in the earth's (inertial) frame during the first half of his journey toward the center of the Milky Way, as a function of Sam's proper time \( \tau \).

3. Using part 2, express earth time \( t \) as a function of Sam's proper time \( \tau \) and find a formula for Sam's distance \( x \) from earth, as measured from earth, in terms of Sam's proper time \( \tau \). Find a formula in terms of inverse hyperbolic functions for \( \tau \) as a function of \( t \) and another formula for \( \tau \) as a function of \( x \).

4. Using part 3, find how many years it takes Sam to go half the distance, 15,000 light years, toward the center of the galaxy. Find the answer both in terms of Sam's proper time \( \tau \) and in terms of earth time \( t \) for this part of the journey (1/4 of the total journey).

5. The two times as measured by Sam and by earth clocks for the entire trip (to the center of the Milky Way and back) can be obtained by multiplying your two answers from part 4 by 4. How much older is Sam by the time he comes back to earth and in what calendar year will he arrive if he begins his journey today?