# Math 595MP
# Spring 2020 Hmwk 8

1) A ball is thrown a horizontal distance of 100 m in 4 seconds at 30° latitude. Assuming a constant speed of the ball, how far is it deflected sideways due to the Coriolis force?

2) The rotation rate of Earth is $\Omega = 7.29 \times 10^{-5}$ in units of $\sec^{-1}$, i.e., radians per second, and Earth's mean radius is 6371 km.

a) Show that the magnitude of the centrifugal acceleration of a point on Earth's surface is $\Omega^2 R$, where $R$ is the perpendicular distance from the point to Earth's axis of rotation, and note that the direction of the centrifugal acceleration is given by the vector $\vec{R}$ from the axis of rotation to the point.

b) Compute the dimensionless parameter $\Omega^2 R/g$. Let $\vec{g}\,'$ denote effective gravity acceleration vector, i.e. the sum of the gravitational acceleration vector $\vec{g}$ (toward the center of the earth) and the centrifugal acceleration vector, $\vec{\Omega} \times (\vec{\Omega} \times \vec{r}) = \Omega^2 \vec{R}$, where $\vec{r}$ is the radial vector from the center of the earth to the point on the surface. Show that the magnitude $||\vec{g}\,'||$ of the effective gravity is about 0.35% less at the equator than at the poles.

c) Show that the maximum angle between $\vec{g}\,'$ and $\vec{g}$ is about 0.1 degrees. At what latitude does the maximum angle occur?

3) Let $\vec{\Omega} = (0, 0, \Omega) \in \mathbb{R}^3$. Find the matrix representation $A_\Omega$ for the linear map $\vec{v} \to \vec{\Omega} \times \vec{v}$. Calculate the exponential $e^{tA_\Omega}$ and show that it equals the rotation matrix $R(t)$ given in lecture.

4) Let $F : \mathbb{R}^3 \to \mathbb{R}$ and $\vec{v} : \mathbb{R}^3 \to \mathbb{R}^3$ be smooth. Prove:

$$\nabla \cdot (F\vec{v}) = \vec{\nabla} F \cdot \vec{v} + F\, \nabla \cdot \vec{v}$$

5) Verify the product rule for the material derivative, i.e.,

$$\frac{D(FG)}{Dt} = F\frac{DG}{Dt} + G\frac{DF}{Dt}$$

6) a) Let $F(x(t), y(t), z(t), t)$ and the indicated functions ($x(t)$, etc.) be smooth and let $g : \mathbb{R} \to \mathbb{R}$ be differentiable. Show that the chain rule for the material derivative in the following sense holds:

$$\frac{D(g(F))}{Dt} = g'(F)\frac{DF}{Dt}$$

b) Using notes from lecture, derive the thermodynamic energy equation for the potential temperature $\theta$:

$$\frac{D\theta}{Dt} = \frac{Q}{c_p}\left(\frac{p_0}{p}\right)^{\kappa},$$

where $Q$ is the diabatic heating rate per unit mass.