Math 483 - Homework 7

Unless otherwise specified in all the problems use Lorenz's original choice of parameters: $\sigma = 10$, $b = 8/3$. You can use the ode45 differential equation solver in Matlab, as illustrated in the example of of the van der pol equation provided in the class webpage.

1) For $r = 28$ Lorenz obtained a chaotic attractor. For $0 < t < 20$ and initial condition $(2, 5, 5)$ investigate the trajectories by using plots of the coordinates vs time, two-dimensional and three-dimensional plots. Find at about what time chaos sets in. Explain your results.

For the case of the chaotic attractor we want a plot which quantifies the exponential separation of the trajectories with time. To do this we must run two calculations with slightly different initial conditions: $(3, 3, 20)$ and $(3 + 10^{-9}, 3, 20)$. If we call the result of the second calculation $\bar{x}$, $\bar{y}$, $\bar{z}$, the goal is to investigate $|\bar{x} - x|$ and a function of t.

If you do this directly, because of the adaptive time stepping in ode45, you will run into the problem that the vectors t for the two runs will be different. To get around this problem, write a new Lorenz program involving six variables instead of three - two completely independent copies of the Lorenz system. Take the initial 6-vector to be $(3, 3, 20, 3 + 10^{-9}, 3, 20)$ and calculate the solution for $0 < t < 20$.

Plot $ln(|\bar{x} - x|)$ vs t. You should get a linear dependence of time which reveals the exponential growth of the separation of the two trajectories with different initial conditions. Use the routine polyfit in Matlab to find the slope of the plot. This gives you an approximate value for the largest Lyapunov exponent for the Lorenz attractor. The three Lyapunov exponents for Lorenz's parameters are $(0.906, 0, -14.572)$. How close is your result? Report your answers with the plot and the fit.

Note that the actual calculation of the Lyapunov exponents is a complex numerical problem. Please see (sprott.physics.wisc.edu/chaos/lyapexp.htm) for a description of the algorithm.

2) For very large $r > 313$ it is possible to show that the solutions become periodic again. Show this for $r = 350$ and your choice of initial conditions and time span.