Matt 396CL Homework #2

1) W+H 3-22

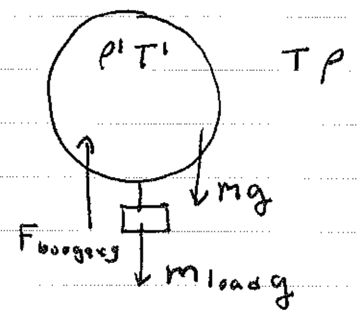
$V_{balloon} = 3000\ m^3$

$m_{load} = 600\ kg$

$T_{ground} = 20°C$

$\Gamma = 0$

$p = 900\ hPa$



If $\Gamma = 0$ $\frac{dT}{dz} = 0$ → Temperature does not change with height. Then $T = 20°C$ at 900 hPa

$$F_{buoyancy} - m_{load}\, g - mg = 0$$

$$\rho V g - m_{load}\, g - \rho' V g = 0$$

$$p = R_d \rho T \qquad p = R_d \rho' T' \quad \text{Since } p' = p$$

$$\rho T = \rho' T' \qquad \rho' = \frac{\rho T}{T'}$$

$$\rho' V = \rho V - m_{load}$$

$$\rho \frac{T V}{T'} = \rho V - m_{load}$$

$$\frac{1}{T'} = \frac{1}{T \rho V}\left(\rho V - m_{load}\right)$$

$$\frac{1}{T'} = \frac{1}{T}\left(1 - \frac{m_{load}}{\rho V}\right) \qquad T' = \frac{T}{\left[1 - \frac{m_{load}}{\rho V}\right]}$$

To get the density of air at 20°C and pressure 500 kPa we must use the information at STP ($T°C$, $p = 1000\,kPa$, $\rho = 1.275\ \frac{kg}{m^3}$)

$$\frac{\rho_1 T_1}{P_1} = \frac{\rho_2 T_2}{P_2} \rightarrow \rho_2 = \rho_1 \left(\frac{T_1}{P_1}\right)\left(\frac{P_2}{T_2}\right)$$

$$\rho = \rho_2 = \left(1.275\ \frac{kg}{m^3}\right)\left(\frac{273}{293}\right)\left(\frac{500}{1000}\right)$$

$$\rho = 1.069\ kg/m^3$$

- Now substitute back into the equation

$$T' = \frac{293\ K}{\left[1 - \frac{800}{(1.069)(3000)}\right]} = 360.4\,K$$

$$T' = 360.4 - 273 = 87.4\ °C$$

2) <u>WHT 3.37</u>

$p = 200\,kPa \quad T = -60°C = -60+273 = 213\,K$

Adiabatic compression to 1000 kPa

$$pV^{\gamma} = \text{constant for adiabatic process} \qquad \gamma = 1.4$$

$$\frac{pV}{T} = \text{constant for ideal gas}$$

$$V \propto \frac{T}{p}$$

$$p\left(\frac{T}{p}\right)^{\gamma} = \text{constant}$$

$$p^{1-\gamma}T^{\gamma} = \text{constant}$$

$$T_2 = \left(T_1^{\gamma}\frac{p_1^{1-\gamma}}{p_2^{1-\gamma}}\right)^{\frac{1}{\gamma}} = T_1\left(\frac{p_1}{p_2}\right)^{\frac{1-\gamma}{\gamma}} = 213\,K\left(\frac{200}{1000}\right)^{\frac{(1-1.4)}{1.4}}$$

$T_2 = 337.3\ K$

$T_2 = 64.4\ °C$

3) W + H 3.41

$q = 0.0196$

$T = 30°C = 303\ K$

$p = 1014\ hPa$

$$q = \frac{m_v}{m_v + m_d} = \frac{w}{1+w} \qquad w = \frac{m_v}{m_d} \quad \text{mixing ratio}$$

• Virtual temperature

$$T_v = T \frac{1}{\left(1 - \frac{e}{p}(1-\varepsilon)\right)}$$

We need to find the vapor pressure e

$$e = \frac{n_v}{n_v + n_d} p = \frac{\frac{m_v}{M_w}}{\frac{m_v}{M_w} + \frac{m_d}{M_d}} p =$$

$$e = \frac{\frac{1}{m_d M_w}}{\frac{1}{m_d M_w}} \left(\frac{m_v}{\frac{m_v}{m_d M_w} + \frac{1}{M_d}} \right) p = \frac{w}{(w+\varepsilon)} p \qquad \varepsilon = \frac{M_w}{M_d} = 0.622$$

→ We can solve for w from the specific humidity

$$q(1+w) = w \qquad q = w(1-q) \qquad w = \frac{q}{(1-q)}$$

$$w = \frac{0.0196}{(1-0.0196)} = 0.02 \leftarrow \text{Appears without units}$$

We must assume the are the same in numerator & denominator

$$w = 0.02 \frac{kg}{kg} = 20 \frac{g}{kg}$$

$$T_v = \frac{T}{\left(1-\left(\frac{w}{w+\varepsilon}\right)(1-\varepsilon)\right)} = \frac{303\ K}{\left(1-\left(\frac{0.02}{0.02+0.622}\right)(1-0.622)\right)}$$

$$T_v = 306.6\ K = 33.6\,^\circ C$$

- Density

$$p = R_d\, \rho_{moist}\, T_v$$

$$\rho_{moist} = \frac{p}{R_d T_v} = \frac{1017 \times 10^2\ \frac{N}{m^2}}{\left(287\ \frac{J}{K}\right)(306.6\ K)} =$$

$$\rho_{moist} = 1.15\ \frac{kg}{m^3}$$

4) W+H 3.42

$$p = 975\ hPa$$

$$T = 15\,^\circ C = 288\ K$$

$$w = 1.8\ \frac{g}{kg} = 1.8 \times 10^{-3}\ \frac{kg}{kg} = 1.8 \times 10^{-3}$$

From the derivation in the last problem

- Vapor pressure

$$e = \frac{w}{w+\varepsilon}\, p = \frac{(1.8 \times 10^{-3})}{(1.8 \times 10^{-3} + 0.622)}(975\ hPa)$$

$$e = 2.81\ hPa$$

- Virtual Temperature

$$T_v = \frac{T}{\left(1 - \frac{e}{p}(1-\varepsilon)\right)} = \frac{288}{\left(1 - \frac{2.81}{975}(1-0.622)\right)} = 288.3\ K$$

$$= 15.3\,^\circ C$$

5) W+H 3.58

In the Carnot Cycle $T_1 = 100°C$, $T_2 = 0°C$

$Q_1 = 20\ J$ in one cycle

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} \longrightarrow Q_2 = Q_1\left(\frac{T_2}{T_1}\right)$$

$$W = Q_1 - Q_2 = Q_1\left[1 - \frac{T_2}{T_1}\right]$$

$$W = 20\ J\left[1 - \frac{273}{373}\right] = 5.36\ J$$ for one cycle

$$W_{total} = 10(5.36) = 53.6\ J$$

6) W+H 3.64

$P_{top} = 600\ hPa$
$P_{bottom} = 1000\ hPa$

$$\alpha_v = \frac{1}{\rho_v} = 1.66\ \frac{m^3}{kg} \qquad \alpha_w = \frac{1}{\rho_w} = 1.00 \times 10^{-3}\ \frac{m^3}{kg}$$

- From eq 3.99

$$\frac{\Delta T_B}{\Delta P_{atmos}} = \frac{T_B(\alpha_2 - \alpha_1)}{L_v} \qquad L_v = 2.25 \times 10^6\ \frac{J}{kg}$$

$$T_B = 100°C = 373\ K$$

$$\Delta T_B = \frac{T_B(\alpha_v - \alpha_w)(P_{top} - P_{bottom})}{L_v}$$

$$\Delta T_B = \frac{(373)(1.66 - 1.00 \times 10^{-3})(600 - 1000) \times 10^2}{2.25 \times 10^6}\ K$$

$$\Delta T = (T_2 - T_1) = (T_2 + 273 - (T_1 + 273))$$

$$\Delta T_B = -11.3\ K = -11.3\ °C$$ (Notice that a difference in temperature is independent of scale)

$$T_B' = T_B + \Delta T_B = 100 - 11.3 = 88.7°C$$