Math 103 Practice Problems

California State University,
Northridge
Department of Mathematics
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The examples in this document are the kinds of problems that may appear on the final exam; many have appeared on previous Math 103 final exams. Some solutions are included to indicate what is expected. Many problems require a summary statement and these are marked with **Sum** . Write your summary statements in complete sentences and include the units of measurement for all quantities mentioned in the summary.

Section 1.1: Linear Equations

1.1.1 The price-demand equation for gasoline is

$$0.2x + 5p = 80,$$

where p is the price per gallon and x is the daily demand measured in millions of gallons.

(a) Write the demand $f(p)$ as a function of price.
$f(p) =$

(b) Sum What is the demand if the price is \$4.00 per gallon? Use the correct units to express your answer.

(c) Sum What price should be charged if the demand is 50 million gallons?

(d) Sum If the price increases by \$0.40, by how much does the demand decrease?

1.1.2 The price-demand equation for gasoline is

$$0.1x + 5p = 40,$$

where p is the price per gallon and x is the daily demand measured in millions of gallons.

(a) Write the demand $f(p)$ as a function of price.
$f(p) =$

(b) Sum What is the demand if the price is \$2.00 per gallon? Use the correct units to express your answer.

(c) Sum What price should be charged if the demand is 100 million gallons?

(d) Sum If the price increases by \$0.10, by how much does the demand decrease?

1.1.3 There is a harvest festival each year on the planet Bozon. During that time, most of the Bozone residents sit down to rejoice and eat roast snig. Past history shows that a reasonable price-demand equation is

$$p = 150 - 2x,$$

where x is the number of kilograms of snig produced and sold, and p is the price per kilogram of snig in the local currency, the Bozat.

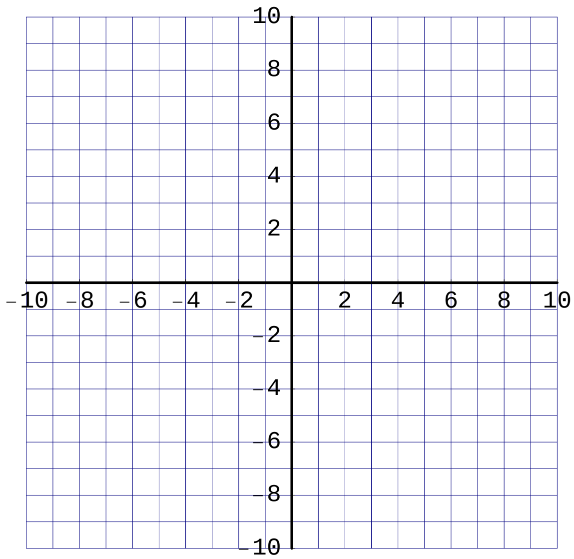
(a) Sum To sell an additional kilogram of snig, how much does price need to decrease?

(b) Solve the price-demand equation for x in terms of p.

(c) Sum What is the revenue if the price is 30 Bozats per kilogram?

Section 1.2: Graphs and Lines

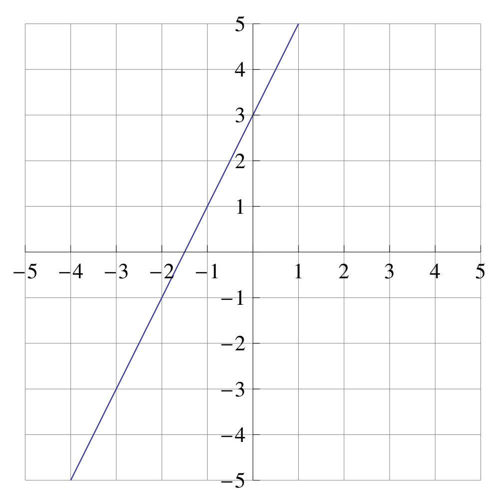
1.2.1 Plot the points $(-7, 2)$, $(2, -4)$ on the graph below and make an **accurate** drawing of the straight line passing through the two points.



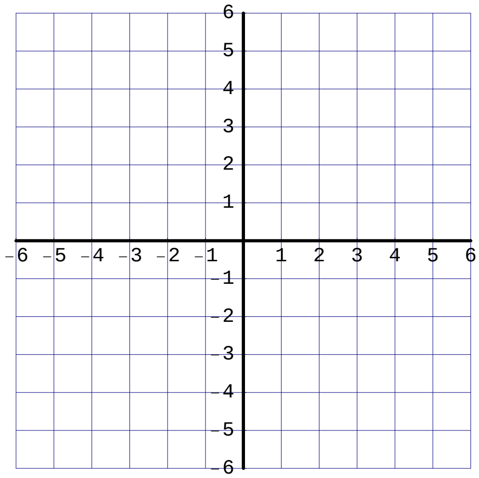
Find the slope of the line:	
What is the y-intercept?	

1.2.2 The graph of a linear function $f(x)$ is shown below.

Find the slope of the line:	
Find $f(-2)$	
What is the y-intercept?	

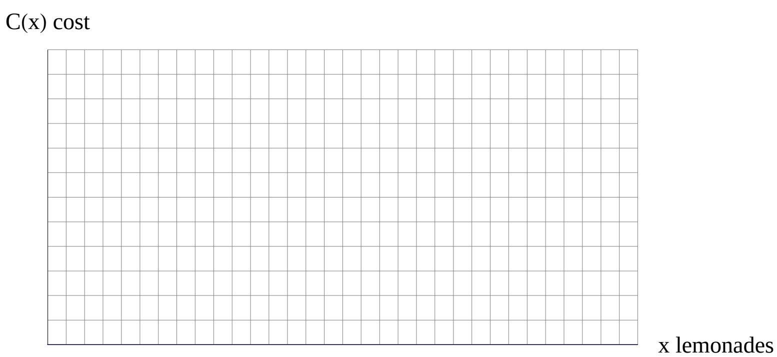


1.2.3 Draw an accurate graph of the function $f(x) = -\frac{2}{3}x+2$. Your graph should clearly show the intercepts, and the point $(3, f(3))$.



1.2.4 Arnold sells lemonade at a school pumpkin fair. It costs him \$3.00 to make 15 lemonades and \$5.00 to make 30 lemonades. The cost, $C(x)$, is a linear function of the number of lemonades x.

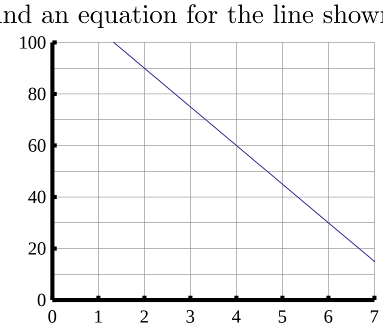
(a) Graph the cost function $C(x)$. Include some of the tick marks on the axes and mark the points corresponding to the cost of 15 and 30 lemonades.



(b) Write an equation for the cost function.

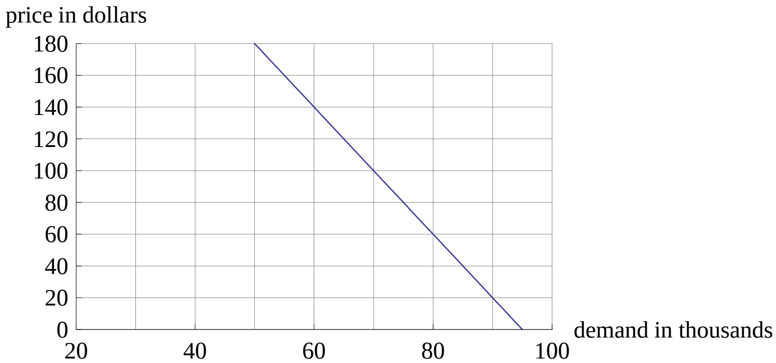
$C(x) =$

1.2.5 (a) Find an equation for the line shown in the graph:



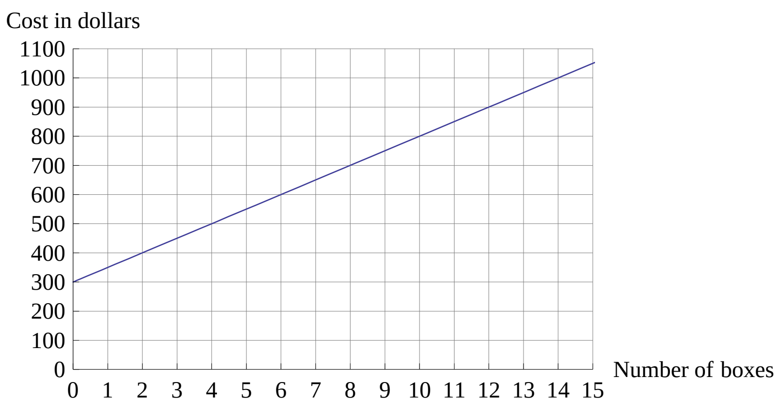
Answer:

(b) The graph of a price-demand equation is shown below. Demand is in thousands of units and price is given in dollars. How many units can be sold at a price of \$60.00?



Number of units:

1.2.6 The graph shows a cost function for producing music boxes. The cost $C(x)$ to produce x music boxes is given in dollars.



(a) What is the cost to produce 11 music boxes?s
Answer only:
Cost to produce 11 boxes:

(b) Sum Find and interpret the slope of the graph of the cost function? (Use the proper units.)

Section 2.1: Functions

2.1.1 Let $f(x) = \dfrac{3x-1}{2x+4}$

(a) Find $f(2)$.

(b) Write the domain of the function in interval notation.

2.1.2 Let

$$f(x) = x^2 - 3x.$$

Evaluate and simplify

$$f(a+1) - f(a).$$

2.1.3 Let $f(x) = 2x^2$. Evaluate and simplify the expression

$$f(3+h) - f(3).$$

2.1.4 Let $f(x) = x^2$. Evaluate and simplify the expression

$$\frac{f(-2+h) - f(-2)}{h}.$$

2.1.5 Let $f(x) = 2x + 3$. Evaluate and simplify the expression

$$f(-1+h) - f(-1).$$

2.1.6 Write the domain of the function $f(x) = \sqrt{x-3}$ in interval notation.

2.1.7 The cost to produce x bookends is

$$C(x) = 240 + 4x,$$

where $C(x)$ is given in dollars.

(a) [Sum] Evaluate and interpret $C(20)$.

(b) [Sum] Write a summary for the statement $C(100) = 640$.

(c) [Sum] What is the cost of producing the 29th bookend?

2.1.8 The cost to produce x doorstops is

$$C(x) = 80 + .40x,$$

where $C(x)$ is given in dollars.

(a) Evaluate and simplify $C(x+1) - C(x)$.

(b) [Sum] What is the cost of producing the 5th door stop?

(c) [Sum] Write a summary for the statement $C(5) = 84$.

2.1.9 A music company sells CDs for a particular artist. The company has advertising costs of \$4000 and recording costs of \$10,000. Their cost for manufacturing, royalties, and distribution are \$5.50 per CD. They sell the CDs to Mega-Mart for \$7.20 each.

(a) Sum What are the fixed costs?

(b) Sum What are the variable costs?

(c) What is the equation for the cost function for x CDs?
$C(x) =$

(d) What is the equation for the revenue function?
$R(x) =$

(e) Sum Write a summary for the statement $R(8000) = 57600$.

(f) Sum How many CDs must the company sell to break even?

2.1.10 Great Neck Pencil Inc. manufactures wooden pencils. The fixed costs for setting up the wood lathes, drills, and yellow paint machine are \$535.00. The variable cost is \$0.12 per pencil.

(a) Write an expression for the cost function $C(x)$, where x is the number pencils manufactured.
$C(x) =$

(b) Sum What is the total cost to manufacture 2,000 wooden pencils?

2.1.11 `[On Webwork]` The price-demand equation for avocados is

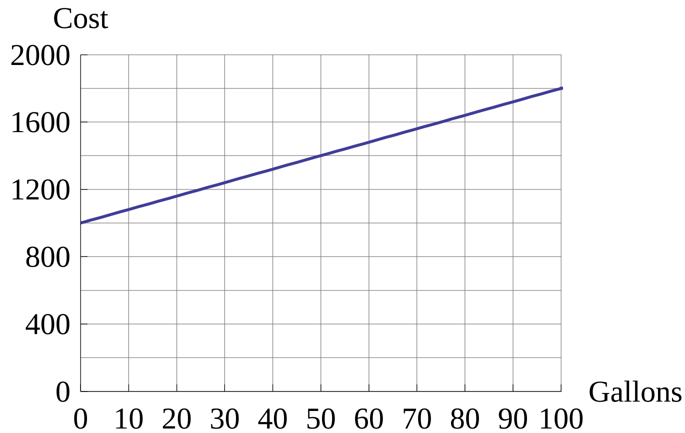
$$20p + x = 25,$$

where p is the price of an avocado and x is the weekly demand (in thousands) for avocados.

Write an expression for revenue as a function of the weekly **demand** for avocados.

$R(x) =$

2.1.12 Slimey Inc. manufactures skin moisturizer. The graph of the cost function $C(x)$ is shown below. Cost is measured in dollars and x is the number of gallons of moisturizer.



(a) Is $C(50) = 1400$?

(b) Sum What are the fixed costs for manufacturing the moisturizer?

(c) What is the slope of the graph of the cost function?

(d) What are the units for the marginal cost?
Circle one

i. dollars
ii. dollars per gallon
iii. gallons
iv. gallons per dollar

2.1.13 The price-demand equation for avocados is

$$20p + x = 25,$$

where p is the price of an avocado and x is the weekly demand (in thousands) for avocados.

Write an expression for revenue as a function of the **price** of an avocado.

$R(p) =$

2.1.14 The DingGnat Doorknob Company intends to sell a new line of square doorknobs. The price-demand function is $p(x) = 45.50 - .06x$. That is, $p(x)$ is the price at which x knobs that can be sold.

b. Sum How many knobs can be sold at a price of \$38.30?

a. Write an equation for the revenue function $R(x)$.

2.1.15 The DingGnat Doorknob Company intends to manufacture a new line of square doorknobs. The company spends \$2,250 dollars in fixed costs to set up the machines and an additional V dollars for each doorknob they make.

a. Write an equation for the cost function $C(x)$, where x is the number of knobs they make.

b. Sum If it costs \$4,850 to make 1000 knobs, what is the variable cost V?.

2.1.16 The DingGnat Doorknob Company intends to manufacture a new line of square doorknobs. The company spends F dollars in fixed costs to set up the machines and an additional \$3.25 for each doorknob they make.

a. Write an equation for the cost function $C(x)$, where x is the number of knobs they make.

b. Sum If it costs \$4,400 to make 100 knobs, what are the fixed costs, F?

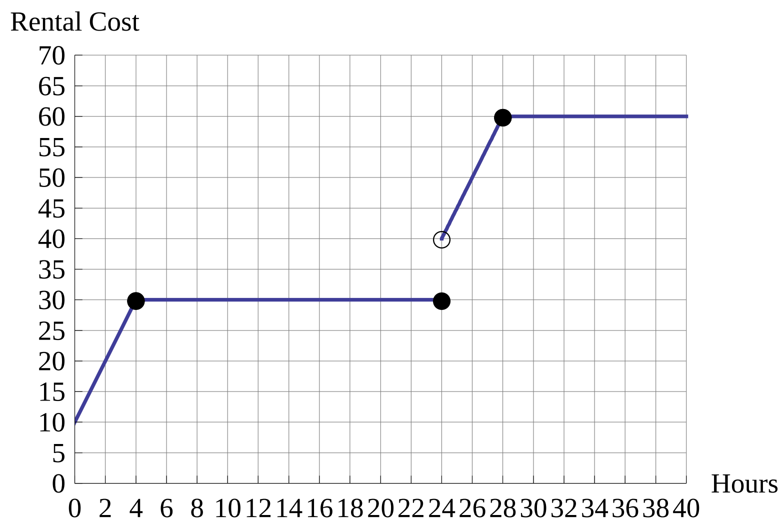
2.1.17 Sum The demand $D(p)$ for StarBoys Frapaccino is function of the price p of a serving. The price p is measured in dollars and the demand $D(p)$ is measured in thousands of servings per day. Translate the symbols $D(2.40) = 171$ into words.

2.1.18 Clippo Inc. manufactures and sells paper clips. The revenue $R(x)$ from the sales of paper clips is a function of how many x they sell. The number sold, x, is measured in thousands of papers clips. The revenue, $R(x)$, is measured in dollars.

a. Sum Write a summary in words for the statement $R(204) = 890$.

b. Clippo management wants to know how many paper clips they must sell to get \$14,000 in revenue. Translate this problem into symbolic form.

2.1.19 At Lake Landloch you can rent boats and the payment follows the graph below.



(a) What is the cost of renting a boat from 8:00am Monday morning to noon on the next day (Tuesday)?

Summary: It costs \$60.

(b) Fill in the blanks in the formula below for the cost $C(x)$ of renting a boat on the domains given below.

$$C(t) = \begin{cases} 10 + 5t, & \text{if } 0 \leq t \leq 4, \\ 30, & \text{if } 4 < t \leq \boxed{24}, \\ \boxed{\text{5t-80}}, & \text{if } \boxed{24} < t \leq 28 \\ 60, & \text{if } t \geq 28 \end{cases}$$

(c) What is the slope of the the graph $y = C(t)$ on the interval $[0, 4]$?

2.1.20 The Trussville Utiliities uses the rates shown in the table below to compute the monthly cost, $C(x)$, of natural gas for residential customers. Usage, x, is measure in cubic hundred feet (CCF) of natural gas.

Base charge	\$6.00
First 800 CCF	\$0.05 per CCF
Over 800 CCF	\$0.10 per CCF

(a) Sum Find the charge for using 200 CCF.

(b) Sum Find an expression for the cost function $C(x)$ for usage under 800 CCF.

(c) Sum Find an expression for the cost function $C(x)$ for usage over 800 CCF.

(d) Write the symbolic form for the statement, "the cost for using 1000 CCF is \$70.00."

2.1.21 The table below shows the electricity rates charged by Madison Utilities in the winter months. Usage, x, is measured in kilowatt hours (KWH) and the charge for using x KWHs is denoted by $C(x)$.

Base charge	\$8.50
First 700 KWH	\$0.06 per KWH
Over 700 KWH	\$0.17 per KWH

a. Sum Find the charge for using 900 KWH.

b. Sum Find an expression for the cost function $C(x)$ for usage under 700 KWH.

c. Sum Find an expression for the cost function $C(x)$ for usage over 700 KWH.

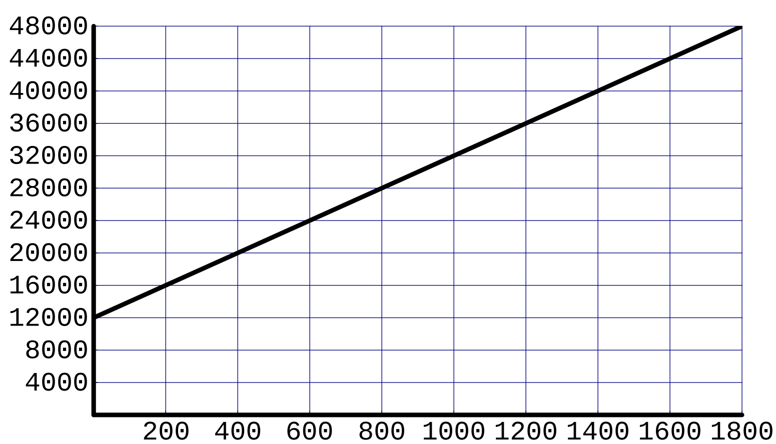
2.1.22 Bigelow Security Inc. is considering producing and selling a new kind of car alarm. The research department estimates that the fixed costs to retool and manufacture the car alarms will be \$12,000 and the variable costs will be \$20 per alarm.

(a) Write an algebraic expression for the total cost to produce x alarms:

Complete Solution:
The fixed costs are 12000 and the variable costs are 20 per unit. So

$$C(x) = 12000 + 20x.$$

(b) Draw an accurate graph of the cost function.

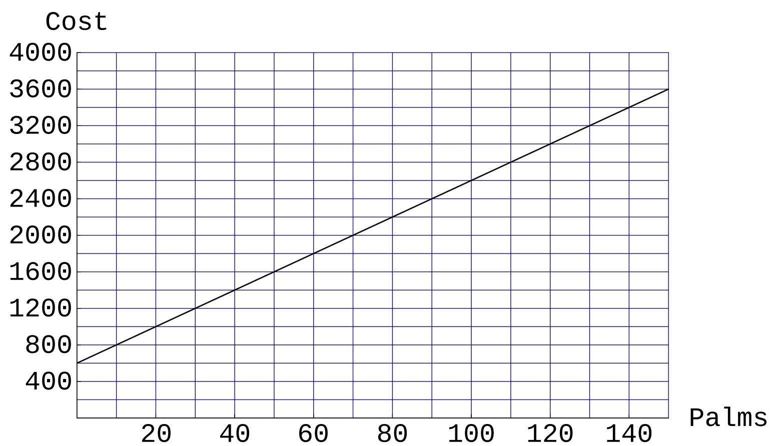


(c) The price demand function of the car alarms is

$$p = 340 - (0.50)x.$$

Price is given in dollars, and x is the demand at price p.
Write an algebraic expression for the revenue function, $R(x)$.
Complete Solution:
$R(x) = xp = x(340 - 0.50x)$.

2.1.23 A nursery grows palm trees from seeds. After a seed has grown for two years, the palm tree is ready to sell. The graph of the cost function is shown below. Cost is given in dollars.

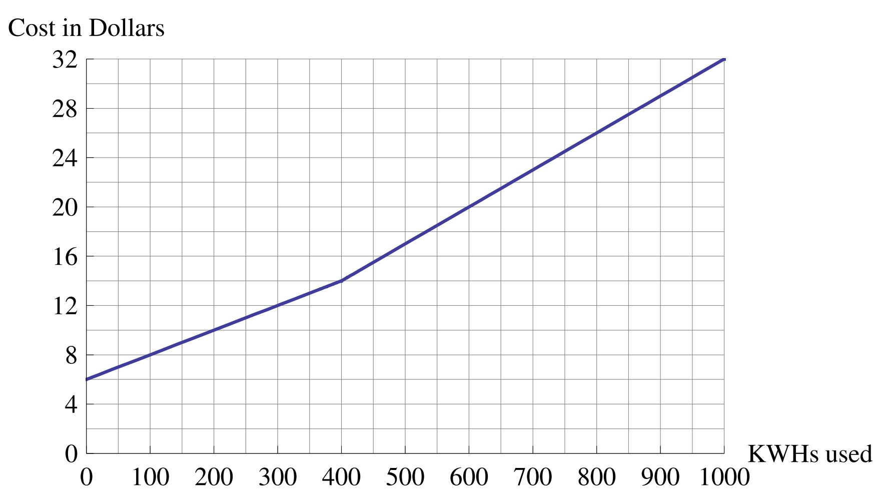


(a) Sum Estimate the cost to grow 100 palms from the graph.

(b) Sum Estimate the average cost (per palm) to grow 100 palms.

(c) Sum Write a summary for the statement $C(50) = 1600$.

2.1.24 The graph shows the monthly cost $C(x)$ for using x kilowatt-hours (KWH) of electricity.



(a) What is the cost for using 200 KWHs?

Answer with units:

(b) What is cost per KWH for a KWH over 400 KWHs per month?

Answer with units:

2.1.25 The price-demand and cost functions for the production and sales of x BBQ grills are given as

$$p(x) = 100 - \frac{x}{2}$$

and

$$C(x) = 500 + 2x.$$

The price per grill $p(x)$ and the cost $C(x)$ are in dollars.

(a) Find the revenue function in terms of x.

Answer:

$R(x) =$

(b) Sum Find the profit earned at an output of 10 BBQ grills.

2.1.26 Suppose a state's income tax code states that the tax liabililty $T(x)$ (in dollars) on x dollars of taxable income is:

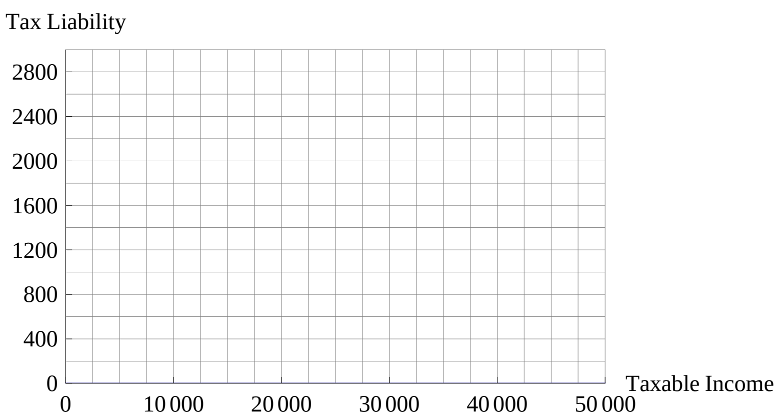
$$T(x) = \begin{cases} 0.04x, & \text{if } 0 < x \leq 20,000 \\ 800 + 0.06(x - 20,000), & \text{if } x > 20,000 \end{cases}$$

(a) Find the tax liablility for taxable incomes of \$20,000 and \$50,000:

Answer only:
Tax on \$20,000:

Answer only:
Tax on \$50,000:

(b) Graph fhe function $y = T(x)$

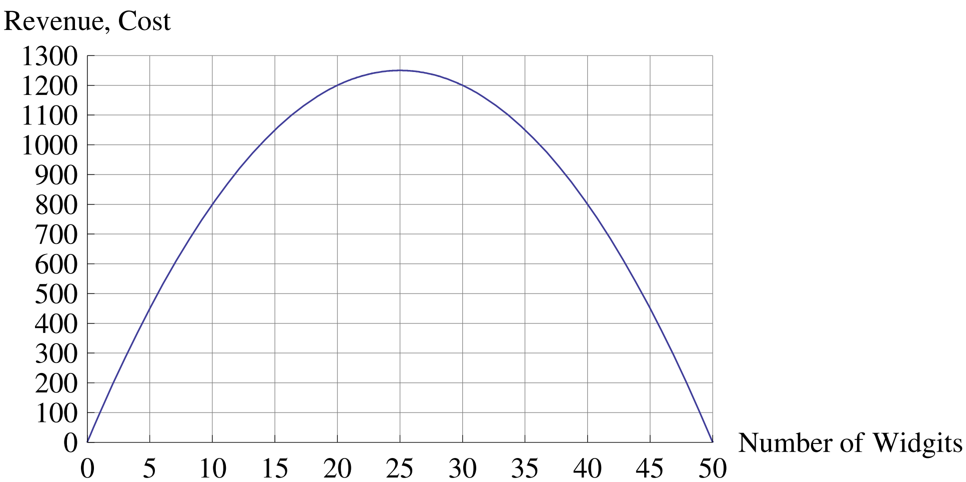


Section 2.2: Elementary functions: Graphs and Transformations

2.2.1 A company manufactures and sells x wigits per week. The weekly price-demand and cost functions are:

$$\begin{aligned} p(x) &= 100 - 2x \\ C(x) &= 700 + 10x. \end{aligned}$$

The revenue function $R(x)$ is graphed below:



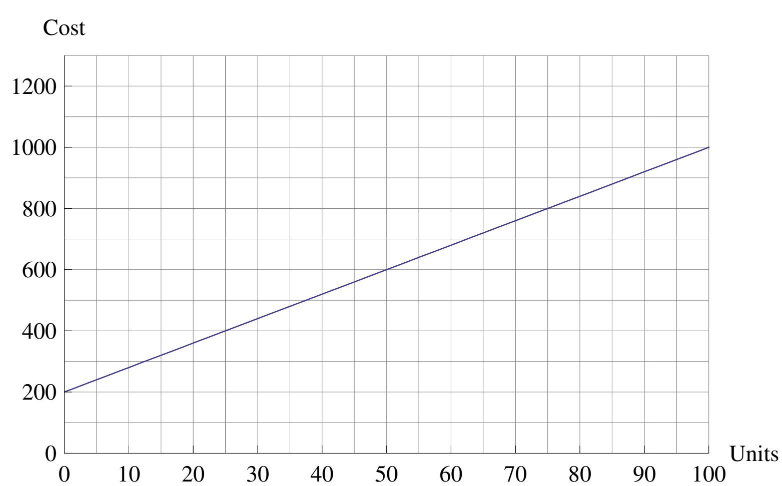
(a) Write an expression for the revenue function, $R(x)$.

(b) Graph the cost function, $C(x)$, on the graph above.

(c) Mark the break-even points on the graph.

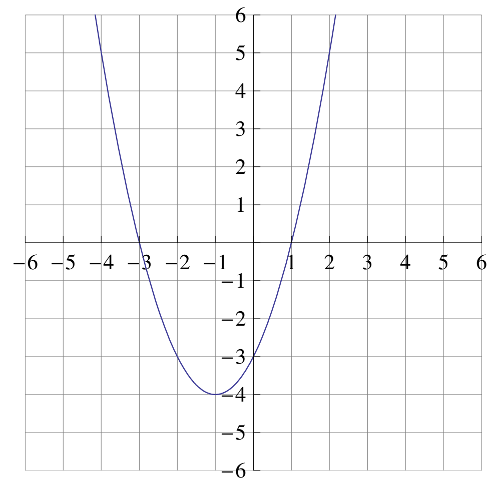
(d) Shade the region where the company makes a profit.

2.2.2 Sum The graph of a cost function is show below. The cost $C(x)$ to produce x units has two parts: the fixed cost F, and the variable (per unit) cost V. Determine the values of F and V from the graph. Explain how you found F and V from the graph.



2.2.3 The graph of a quadratic function $f(x)$ is shown below.

Find the y-intercept	
Find the x-intercepts	
Find $f(-2)$	



2.2.4 Relative to the graph of

$$y = 2x^2 - 1,$$

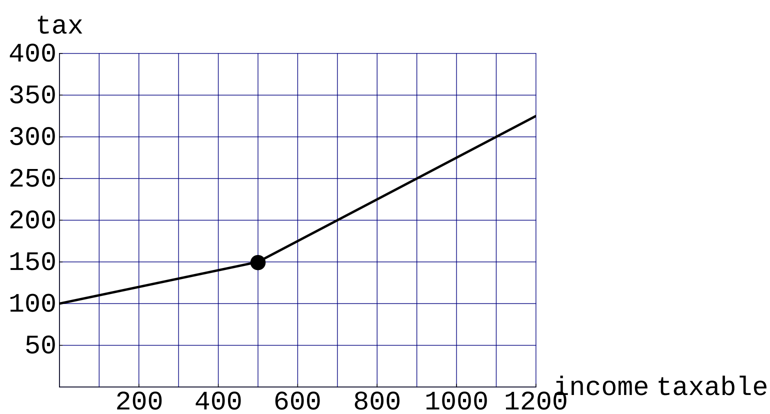
the graphs of the following equations have been changed in what way?

Answer
↓

	1. $y = 2x^2 + 4$
	2. $y = 2(x + 5)^2 - 1$
	3. $y = 10x^2 - 5$

A	shifted 5 units right
B	shifted 5 units left
C	stretched vertically by a factor of 5
D	shrinked vertically be a factor of 1/5
E	shifted 5 units up
F	shifted 5 units down

2.2.5 The following graph shows the amount of tax $T(x)$ (in dollars) for a taxable income of x (in dollars).



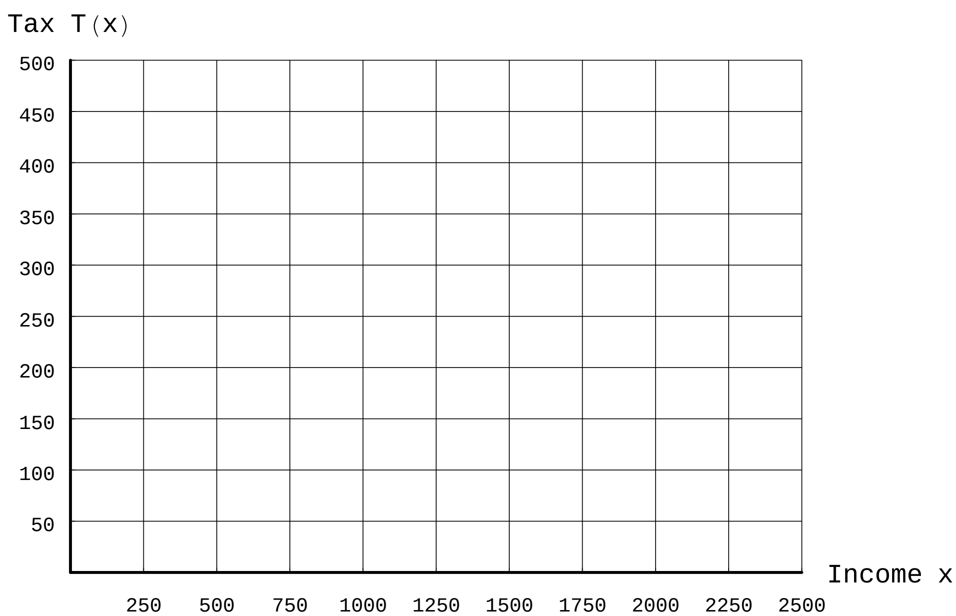
Sum What is the tax rate for incomes over \$500? Give your answer as a percentage and explain how you calculated it.

2.2.6 There is an income tax on the planet Bozone. Both annual income, x, and income tax, $T(x)$, are measured in the local currency, the Bozat ($\mathfrak{B}$). If the annual income $x < 1500$, then the income tax is 10% of the income: $T(x) = .10x$. If annual income $x \geq 1500$, then the income tax is 20% of the income: $T(x) = .20x$.

(a) The equation for the income tax on income between $\mathfrak{B}\,0$ and $\mathfrak{B}\,1500$ is of the form $T(x) = mx+b$. Find the values of m and b.

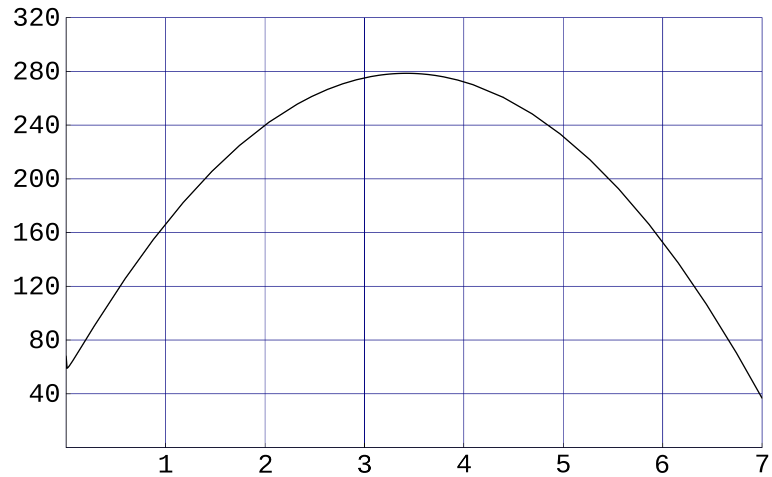
(b) The equation for the income tax on income above $\mathfrak{B}\,1500$ is of the form $T(x) = mx + b$. Find the value of m.

(c) Draw an accurate graph of the tax function $T(x)$.

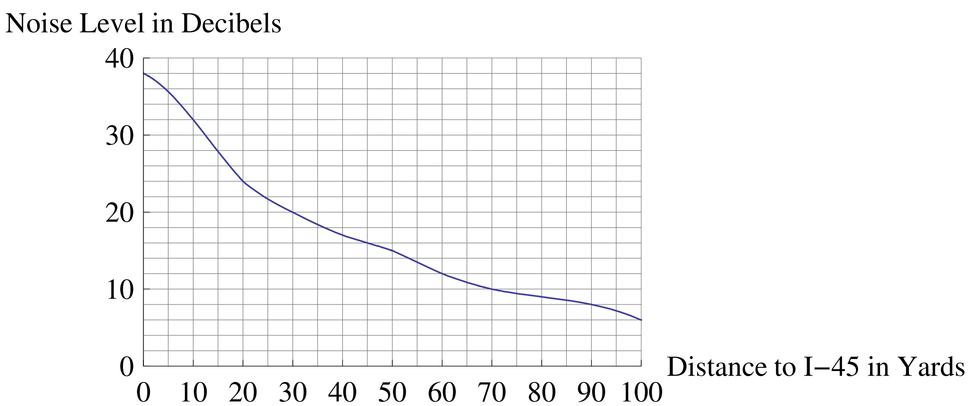


(d) Sum Is the income tax function, $T(x)$, continuous? Explain.

2.2.7 The graph below shows the amount of $A(t)$ electricity used in the city of Roseville as a function of the time of day t. The unit of measurement for electricity is megawatts and the time t is the number of hours past noon. How much electricity is being used at 2:00pm?



2.2.8 Ozone Avenue runs east-west and crosses under Interstate I-45 in town of Carbonia. The noise level of I-45 along Ozone Avenue is shown in the graph below. Noise level is measured in decibels and the distance along Ozone Avenue from the underpass is measures in yards. $N(x)$ is the noise level x yards from the underpass.



(a) What is the noise level 70 yards from the underpass?

(b) At what point on Ozone Avenue is the noise level 18 decibels?

(c) Sum Give a summary of the statement $N(90) = 8$.

(d) Sum The noise level 45 yards from I-45 is 26 decibels. If you move 25 yards farther away from I-45, does the noise level increase or decrease? by how much?

Section 2.3: Quadratic Functions, General Polynomials, Rational Functions

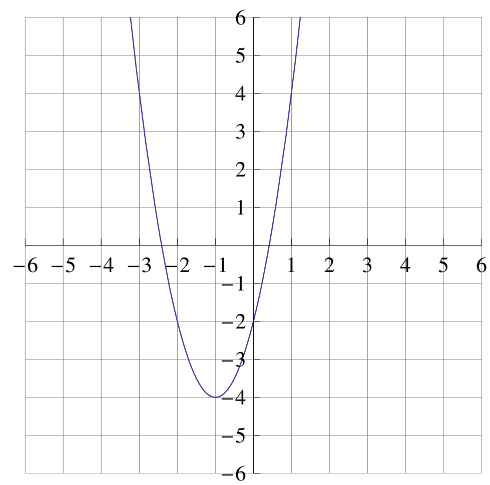
2.3.1 Let $f(x)$ be the quadratic function:

$$f(x) = 3x^2 + 6x - 1.$$

Write $f(x)$ in the vertex-form $f(x) = a(x-h)^2 + k$.	
What are the coordinates of the vertex of the parabola?	
Find the y-intercept	

2.3.2 The graph of a quadratic function $f(x)$ is shown below.

Find the vertex of the parabola:	
Find $f(-3)$	
Write an equation for $f(x)$	
$f(x) =$	



2.3.3 Let $f(x)$ be the quadratic function:

$$f(x) = -x^2 - 6x + 2.$$

(a) By completing the square, write $f(x)$ in the vertex-form.

(b) What is the vertex of the parabola?

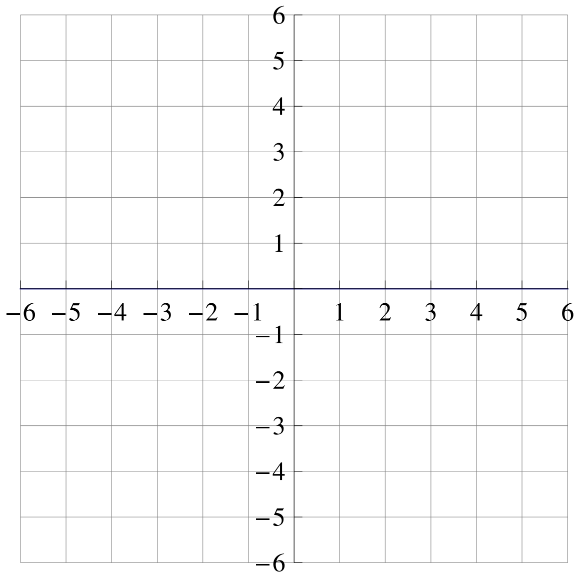
(c) What is the maximum or minimum value of the function?

(d) What is the range of the function?

(e) What is the y-intercept?

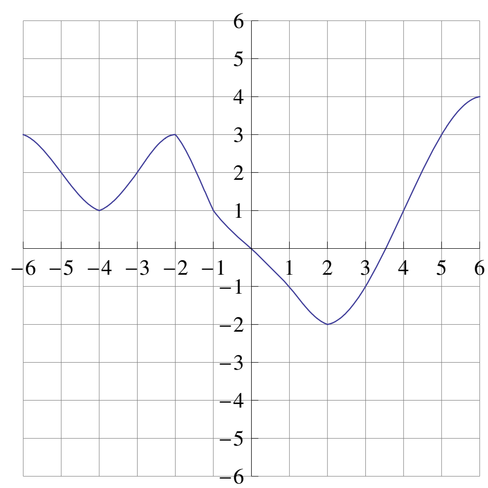
(f) Does the parabola have one, two, or no x-intercepts?

2.3.4 Draw an accurate graph of the function $f(x) = \dfrac{4x}{2x+1}$. Your graph should clearly show the asymptotes, the point $(-1, f(-1))$, and the y-intercept.



2.3.5 Find the coordinates of the turning points in the graph below. Identify each turning point as either a local maximum or a local minimum.

Turning point coordinates	Local max or min?



2.3.6 Consider the polynomial function

$$f(x) = 4x^5 + 8x^4 - 64x^3 - 56x^2 + 252x - 144.$$

(a) What is the degree of this polynomial?

(b) What is the maximum number of times this polynomial can intersect the x-axis?

(c) What is the maximum number of turning points this polynomial can have?

2.3.7 `[Done in 103L, variant on webwork.]` Relative to the graph of

$$y = \frac{1}{x+3},$$

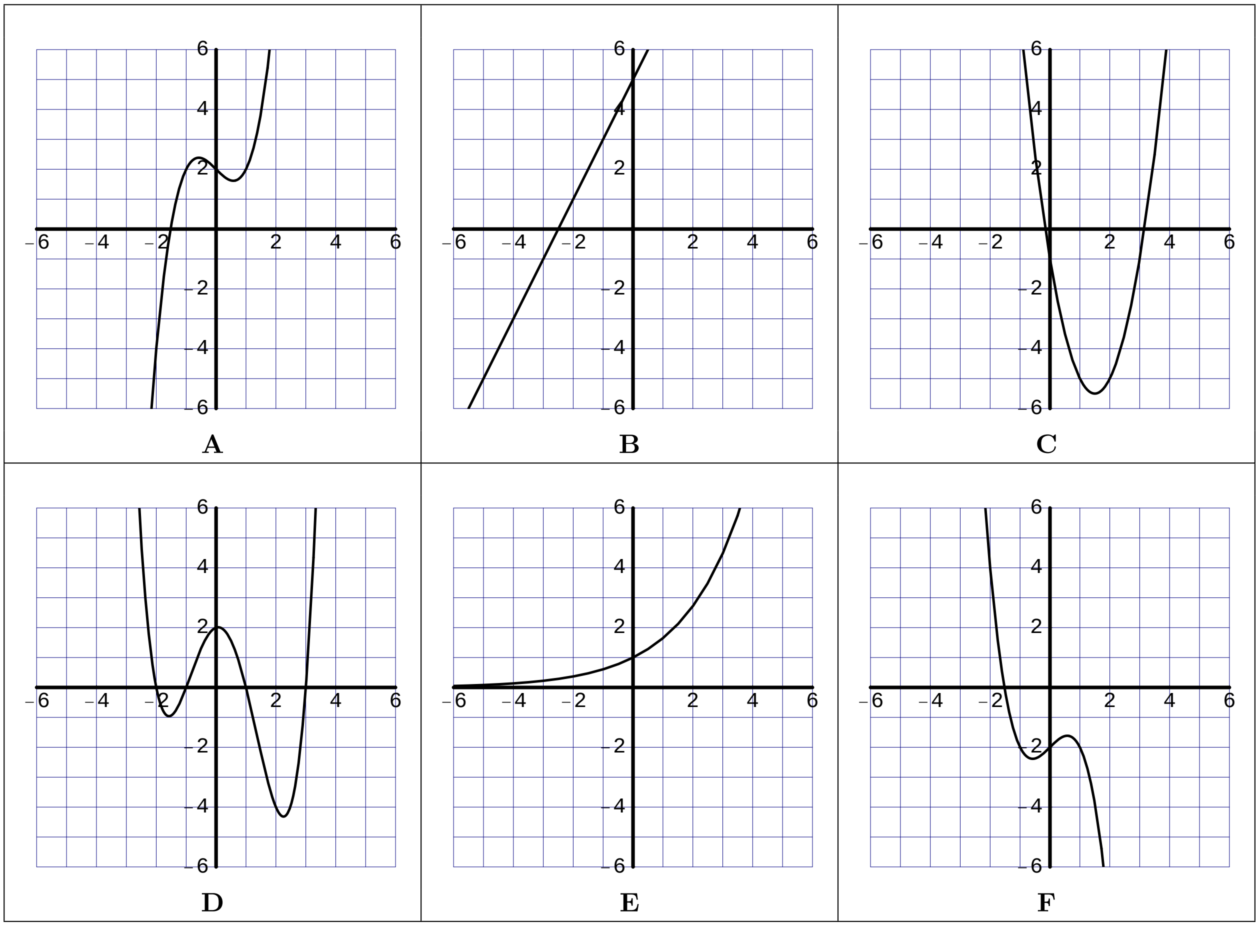
the graphs of the following equations have been changed in what way?

Answer
↓

	1. $y = \dfrac{1}{5(x+3)}$
	2. $y = \dfrac{1}{(x+3)} + 5$
	3. $y = \dfrac{1}{x+6}$

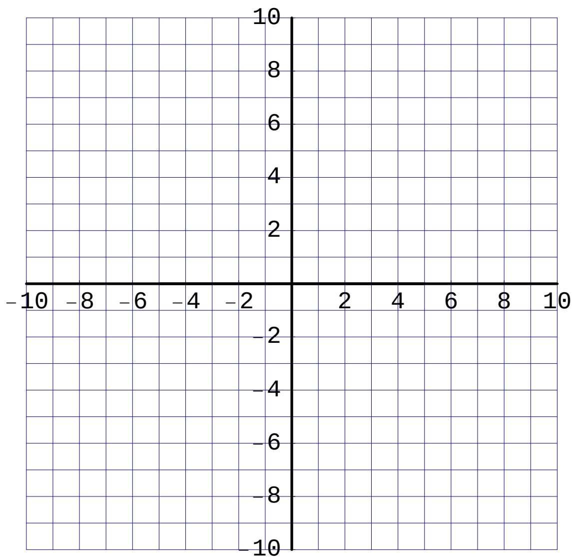
A	shifted 5 units right
B	shifted 5 units left
C	stretched vertically by a factor of 5
D	shrinked vertically be a factor of 1/5
E	shifted 5 units up
F	shifted 5 units down

2.3.8 Match the graph with function:

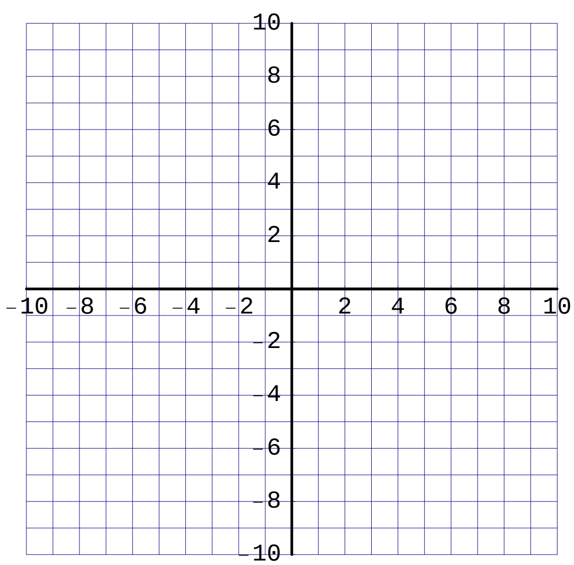


Graph	Function
	$f(x) = 2x^2 - 6x - 1$
	$f(x) = -x^3 + x - 2$
	$f(x) = (1/3)(x-1)(x+2)(x-3)(x+1)$
	$f(x) = x^3 - x + 2$

2.3.9 Make an **accurate** graph of the function $f(x) = -(x+2)^2 + 1$. Mark the y-intercept, the vertex, and the points $(-1, f(-1)), (-3, f(-3))$ with dots on the graph.



2.3.10 Make an **accurate** graph of the quadratic function $f(x) = -x^2 + 5$. Mark the y-intercept, the vertex, and the points $(1, f(1)), (-2, f(-2))$ with dots on the graph.

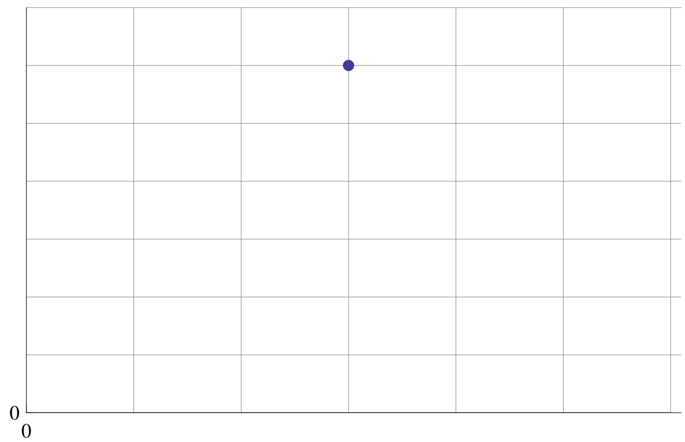


2.3.11 Let $f(x)$ be the quadratic function:

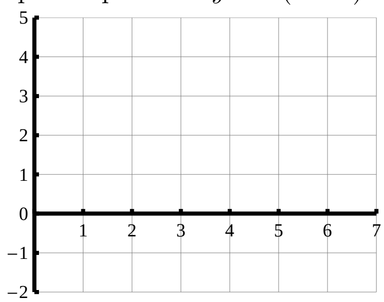
$$f(x) = 2x^2 - 6x + 1.$$

	Answer
Write $f(x)$ in the vertex-form $f(x) = a(x-h)^2 + k$.	
What are the coordinates of the vertex of the parabola?	
Does the parabola open up or down?	

2.3.12 The vertex of the graph of $y = 240x - 4x^2$ is shown as a dot on the graph paper below. Make an accurate drawing of the parabola showing the scales on the x- and y-axes, the x-intercepts, and the coordinates of the vertex.



2.3.13 Graph the parabola $y = -(x-3)^2 + 4$. Mark the vertex, and the x-intercepts on the graph.



2.3.14 The range of the rational function $f(x)$ includes all but one number. What is that number? Prove that the number is not in the range of $f(x)$.

$$f(x) = \frac{2x-1}{5x+1}.$$

2.3.15 Let $f(x) = \dfrac{3x-1}{2x+4}$ Prove that $y = \frac{3}{2}$ is not in the range of $f(x)$.

Section 3.123: Mathematics of Finance

3.123.1 In the problems below please simplify your answers as far as possible without a calculator. You may leave your answers in terms of exponentials and logarithmic expressions.
Suppose we deposit \$20,000 into an investment account.

(a) Sum How much will our account have after 12 years if it is invested at an annual interest rate of 3% compounded every four months?

(b) Sum How long will it take for the investment account to grow to \$100,000 if annual interest is 3% and it is compounded continuously?

3.123.2 Suppose we deposit \$7000 into an investment account. Simplify you answers as far as possible without a calculator. You may leave your answers in terms of exponentials and logarithmic expressions.

(a) What amount will our account have after 15 years if it is invested at an annual rate of 5% compounded quarterly.
Complete Solution:
We use the formula for future value:

$$A = P(1+i)^n,$$

where $P = 7000, i = .05/4, n = 15 \times 4$.

$$A = 7000(1.0125)^{60}.$$

Summary: The account will have $7000(1.0125)^{60}$ dollars after 15 years.

(b) What annual rate of interest is needed in order for the investment account to grow from \$7000 to \$14,000 in 10 years if interest is compounded continuously?

Complete Solution:
We use the formula for continuous compounding:

$$A = Pe^{rt},$$

where $P = 7000, A = 14000, t = 10$ and r is unknown. Thus we must solve

$$14000 = 7000e^{10r}$$

for r:

$$\begin{aligned} 14000 &= 7000e^{10r} \\ e^{10r} &= 2 \\ 10r &= \ln 2 \\ r &= \frac{\ln 2}{10}. \end{aligned}$$

Summary: For an investment of \$7000 to grow to \$14000 in 10 years with continuous compounding, the rate must be $r = (\ln 2)/10$.

3.123.3 Sum We deposit \$2,000 into an account earning 6% interest compounded semiannually. How many years will it take for the account grows to \$5,000?

3.123.4 In the problems below you may leave your answers in terms of exponentials and logarithmic expressions.

(a) Sum Suppose we deposit \$3,000 into an investment account. How much will our account have after 8 years if it earns interest at an annual interest rate of 4% compounded continuously?

(b) Sum Suppose we deposit \$3,000 into an investment account. How long will it take for the investment account to grow to \$30,000 at an annual interest rate of 4% compounded quarterly?

3.123.5 In the problems below please simplify your answers as far as possible without a calculator. You may leave your answers in terms of exponentials and logarithmic expressions.

(a) Suppose you deposit \$7,000 into an investment account earning 8% interest compounded quarterly. How much will your account have after 30 years?

Answer with units:

(b) Suppose \$7,000 is deposited into an investment account with interest compounded continuously. The amount triples in 20 years. What is the interest rate?

Answer as a PERCENT:

3.123.6 In the three problems below, you may write your answer in terms of exponents and/or logarithms.

(a) Sum Alex deposits \$1200 into an investment account earning interest at an annual rate of 8% compounded quarterly. How much will his account have after 5 years?

(b) Sum Maria deposits \$2000 into a bank account earning interest at an annual rate of 7% compounded continuously. How long will it take for her account to grow to \$3000?

(c) Sum Felix wants to buy a new high-def TV that costs \$2000. How much does he have to invest now to have \$2000 in three years, if his investment earns 8% compounded annually?

3.123.7 Please simplify you answers as far as possible without a calculator. You may leave your answers in terms of exponentials and logarithmic expressions.

(a) If \$30,000 is deposited in an account at an annual rate of 8% compounded quarterly, how much will be in the account after 15 years?

Answer with units:

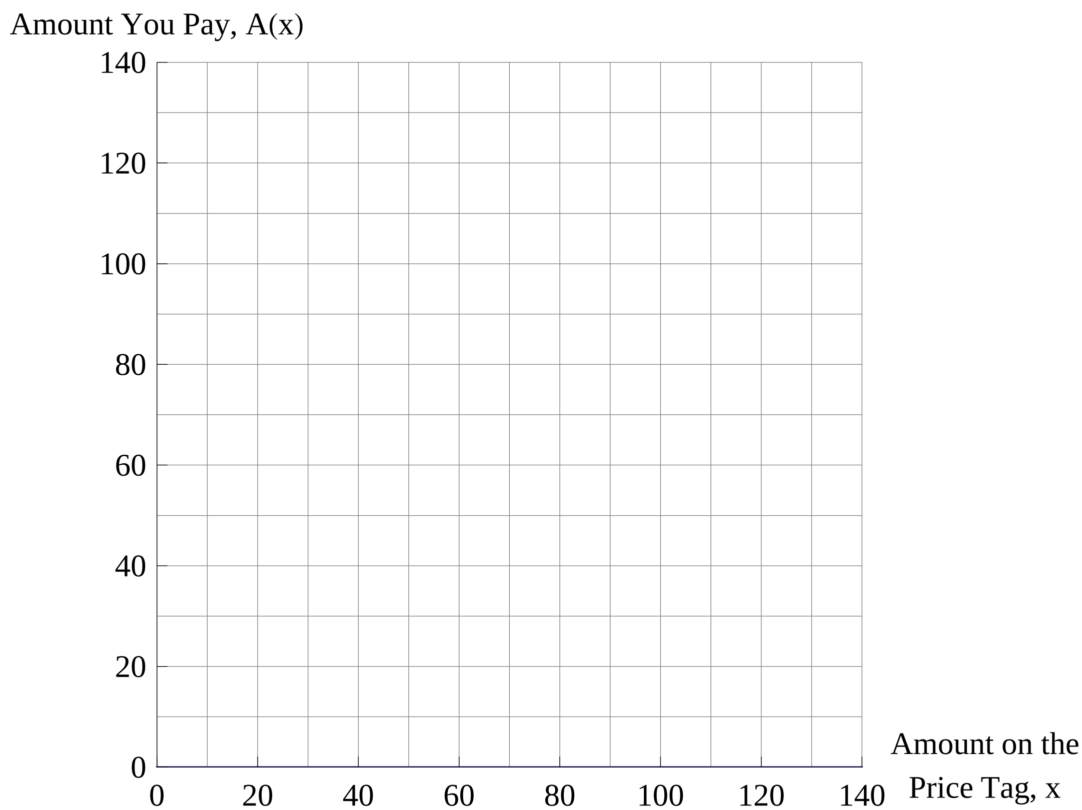
(b) How long will it take for an investment of \$40, 000 to grow to \$120,000 at the rate of 5.75% compounded continuously?

Answer with units:

Section 10.12: Limits, Continuity

10.12.1 Inkko Office Supplies is going out of business. To stimulate sales, Inkko will give its customers a \$10.00 reduction on any item with a price tag **over** \$50.00. For example, if you buy a paper shredder with a price tag of \$55.00, you pay only \$45.00. But if the price tag is \$50.00, you pay \$50.00. If you buy a box of copy paper with a price tag of \$24.99, you pay \$24.99.

(a) Draw the graph of the amount you pay, $A(x)$ as a function of the amount on the price tag, x.



(b) What is the limit $\lim_{x\to 50^+} A(x)$?

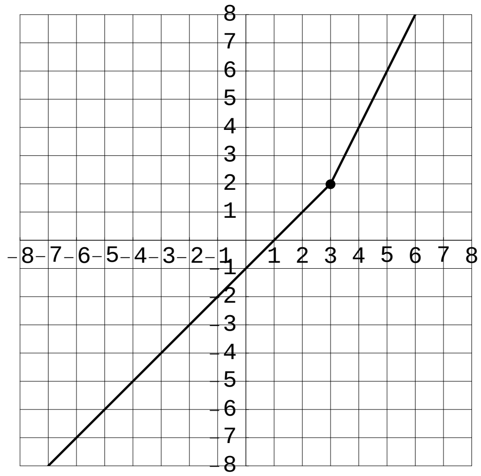
circle one:

40	50	does not exist	$x = 10$

10.12.2 Consider the following function.

$$f(x) = \begin{cases} x-1, & \text{if } x \le 3 \\ 2x-4, & \text{if } x > 3 \end{cases}$$

(a) Sketch a graph of $y = f(x)$.
Complete Solution:



(b) Where is this function continuous? Explain why using limits.
Complete Solution:
The function is continuous at every value of x. The only possible exception is at $x = 3$ and at that point,

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} x - 1 = 2,$$

and

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} 2x - 4 = 2.$$

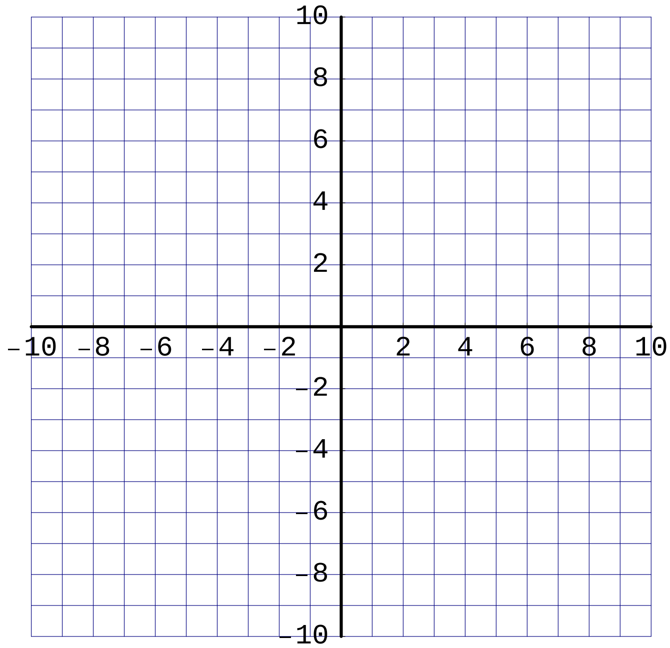
So $\lim_{x \to 3} f(x)$ exists and equals the value of the function $f(3) = 2$.

(c) Where is this function differentiable? (No explanation necessary)
Complete Solution:
The function is differentiable at every value of x except at $x = 3$. The function is not differentiable at $x = 3$.

10.12.3 Let $f(x)$ be the following piecewise defined function:

$$f(x) = \begin{cases} -2x + 1, & \text{if } x \leq 2 \\ 3, & \text{if } x > 2 \end{cases}$$

(a) Graph the function $y = f(x)$.



(b) Find $\lim_{x\to 2^+} f(x)$.

(c) Find $\lim_{x\to 2^+} f(x)$.

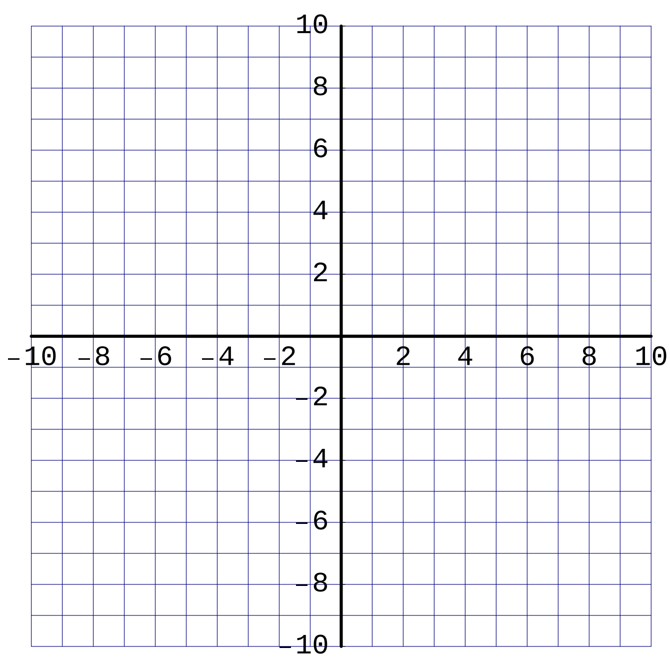
(d) Where is this function continuous?

(e) Where is this function differentiable?

10.12.4 Let $f(x)$ be the following piecewise defined function:

$$f(x) = \begin{cases} \frac{x}{2} + 5, & \text{if } x < -2 \\ x, & \text{if } x \geq -2 \end{cases}$$

(a) Graph the function $y = f(x)$.



(b) Find $\lim_{x\to -2^+} f(x)$.

(c) Find $\lim_{x\to -2^+} f(x)$.

(d) Where is this function continuous?

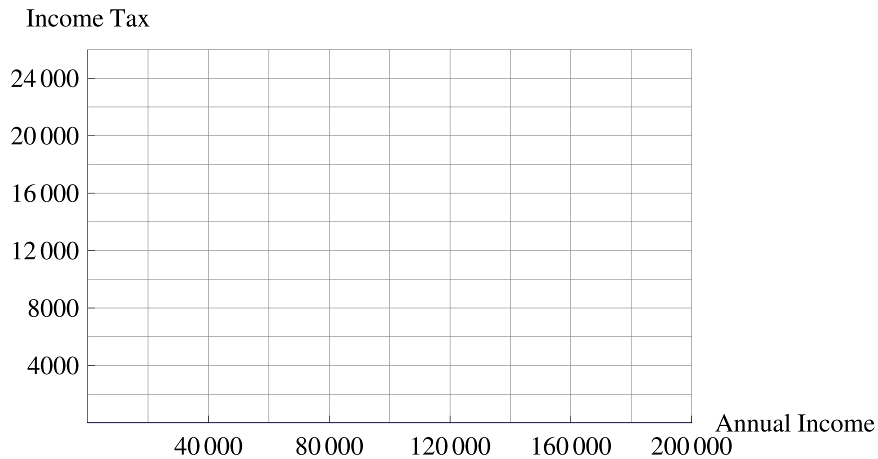
(e) Where is this function differentiable?

10.12.5 Suppose that Alaska decides to enact an income tax using the income tax table shown below. Both annual income, x, and income tax, $T(x)$, are measured in dollars.

Annual Income	Base Tax	Rate	Of the Amount Over
From \$0 to \$120,000	0	10%	\$0
Over \$120,000	\$12,000	25%	\$120,000

(a) Sum What is the income tax for an annual income of \$160,000?

(b) Draw an accurate graph of the income tax function $T(x)$. Mark the points on the graph corresponding to annual incomes of \$120,000 and \$160,000 with their coordinates.



(c) Is the tax function $T(x)$ continuous at $x = 12,000$? Discuss in terms of tax policy.

Section 10.3: Infinite limits and limits to infinity

10.3.1 Evaluate $\lim_{x\to\infty} \frac{7x+3}{14x-3}$.

10.3.2 Evaluate $\lim_{x\to\infty} \frac{6x+3}{3x-1}$.

10.3.3 Evaluate $\lim_{x\to 2^-} \frac{7x+3}{5x-10}$.

10.3.4 Evaluate $\lim_{x\to 2^+} \frac{7x+3}{5x-10}$.

10.3.5 Evaluate $\lim_{x\to 10^-} \frac{900x+30,291}{30x-300}$.

10.3.6 Evaluate $\lim_{x\to 10^+} \frac{900x+30,291}{30x-300}$.

10.3.7 Evaluate $\lim_{x\to\infty} x^3+3x-1$.

10.3.8 Evaluate $\lim_{x\to\infty} -x^2+1000x-10,000$.

Section 10.4: The Derivative

10.4.1 Consider the revenue function $R(x) = 120x - 0.5x^2$ for selling x widgets.

(a) Sum Find the change in revenue when sales change from $x = 4$ to $x = 6$.

(b) Sum Find the average rate of change of revenue for this change in sales levels.

(c) Sum Use this to estimate the revenue at a production of $x = 6$.

10.4.2 The profit (in dollars) from the sale of x palm trees is given by

$$P(x) = 20x - 0.01x^2 - 100.$$

(a) Sum Find the average change in profit if sales changes from 10 trees to 11 trees.

(b) Sum Find the profit and the instantaneous rate of change of profit at a sales level of 10 trees.

10.4.3 Use the definition of the derivative to find the derivative of $f(x) = 2x^2 - 3$. Here are some steps.

(a) Find

$$f(x+h)$$

(b) Find

$$\frac{f(x+h) - f(x)}{h}$$

(c) Find

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

10.4.4 The revenue from the sale of x cellphone towers is given by

$$R(x) = 1800x - 5x^2.$$

(a) Sum What is the change in revenue if production is changed from $x = 12$ to $x = 13$ cellphone towers?

(b) What is the (instantaneous) rate of change in revenue at $x = 12$?

10.4.5 The revenue from the sale of x high end cameras is given by

$$R(x) = 1000x - 2x^2.$$

(a) Sum What is the change in revenue if production is changed from $x = 10$ to $x = 11$ cellphone towers?

(b) What is the (instantaneous) rate of change in revenue at $x = 10$?

10.4.6 Find the derivative $f'(x)$ of the function

$$f(x) = (2x^3 + 5)^{10}.$$

$f'(x) =$

10.4.7 Let $g(x) = x^3 + 2x + 5$. Find an equation for the line tangent to the graph of $g(x)$ at the point $(0, g(0))$.

Equation of tangent line:

10.4.8 Often the sales of Kobe Bryant's new pairs of basketball shoes increases and then levels off. The monthly sales $S(t)$ (in thousands of shoes) as a function of the number of months t since the shoes first came on the market is given by :

$$S(t) = \frac{90t^2}{t^2 + 50}.$$

(a) Sum Find and interpret $S(10)$.

(b) Find and simplify the derivative of $S(t)$.

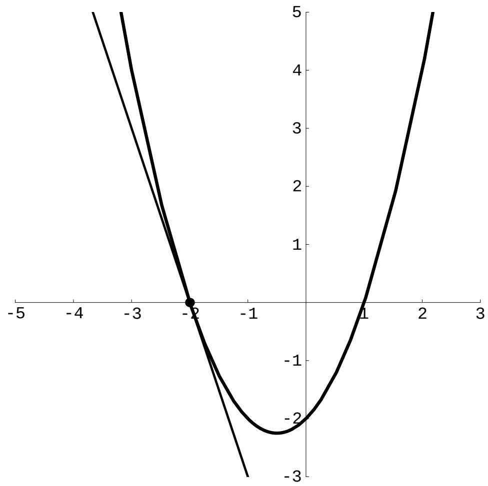
Answer only:
$S'(t) =$

(c) Find and interpret $S'(10)$.

Section 10.5: Basic Derivatives

10.5.1 Let $f(x) = -2x^2 + x + 1$. Find the equation of the line tangent to the graph of $y = f(x)$ at the point $(-3, f(-3))$.

10.5.2 Let $f(x) = x^2 + x - 2$. Find the equation of the line tangent to the graph of $f(x)$ at the point $(-2, f(-2))$ shown below.



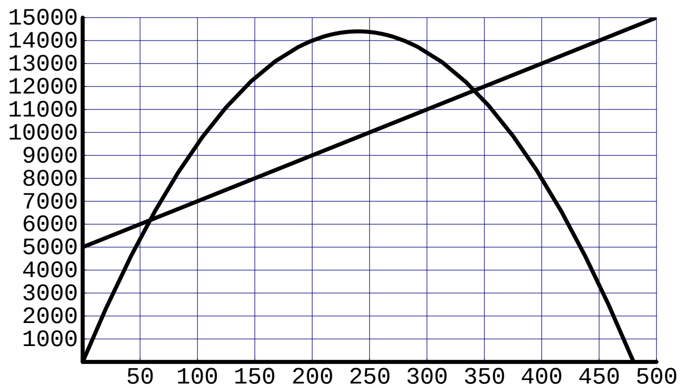
10.5.3 Let $f(x) = -x^2 - 4x$. Find the equation of the line tangent to the graph of $f(x)$ at the point shown $(2, f(2))$.

10.5.4 Find the derivative of

$$f(x) = \frac{2x^3 - x^2 + 4x + 1}{x}.$$

Section 10.7: Marginal Analysis

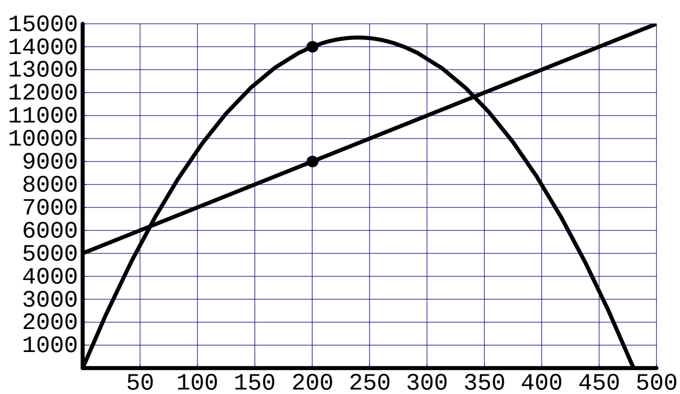
10.7.1 The graphs of the revenue and cost functions for the production and sale of x units are shown below. The cost function is the straight line and the revenue function is the curve.



(a) Use the graph to estimate the production level x that maximizes the profit.

(b) Mark the points $(x, C(x))$ and $(x, R(x))$ on the graphs of the cost and revenue functions corresponding to the value of x that maximizes profit.

Complete Solution:
The maximum profit occurs at the production level x where marginal revenue equals marginal cost. That is, where the slopes of the revenue and cost curves are equal. The slopes are equal at $x = 200$. The points $(200, C(200)) = (200, 9000)$ and $(200, R(200)) = (200, 14000)$ are marked below.



Summary: Profit is maximized when the production level is 200 units.

(c) What is the maximum profit?

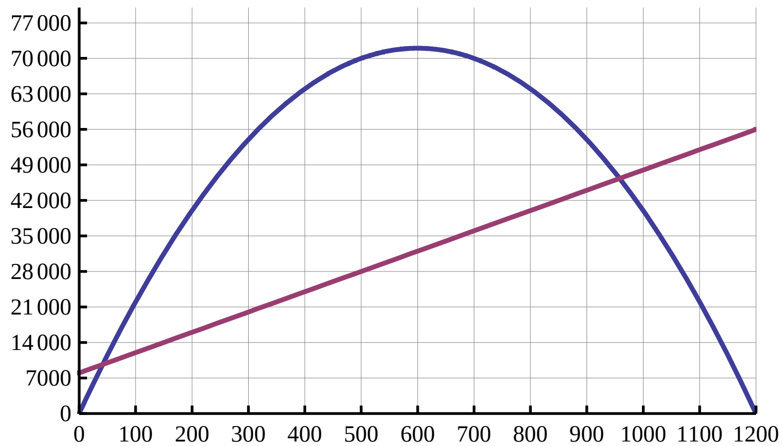
Complete Solution:
Profit is the difference between revenue and cost. $R(200) - C(200) = 14000 - 9000 = 5000$.

Summary: The maximum profit is \$5000.

(d) If the cost per unit decreases, should the production level be raised or lowered to maximize profit? Explain in terms of the graph.

Summary: If the cost per unit decreases, then the slope of the cost curve decreases. The slope of the revenue curve decreases as production level increases. So production level must be increased (raised) to maximize profit.

10.7.2 The graphs of the revenue and cost functions for the production and sale of x units are shown below. The cost function is the straight line and the revenue function is the curve.



(a) Use the graph to estimate the production level x that maximizes the profit.

(b) Mark the points $(x, C(x))$ and $(x, R(x))$ on the graphs of the cost and revenue functions corresponding to the value of x that maximizes profit.

(c) What is the maximum profit?

(d) If the fixed costs increase by ten dollars, should the production level be raised, lowered, or remain the same to maximize profit? Explain in terms of the graph.

10.7.3 Sequoia Publishing Company plans to publish a vegetarian cookbook. The cost (in dollars) to produce x books is

$$C(x) = 2600 + 10x.$$

The price-demand equation is

$$p = 80 - (0.14)x$$

and the revenue function is

$$R(x) = 80x - (0.14)x^2.$$

(a) Compute the marginal cost and marginal revenue functions:

Complete Solution:

Marginal cost: $C'(x) = 10$

Marginal revenue: $R'(x) = 80 - 0.28x$

(b) Use the marginal revenue function to approximate the revenue for selling the 201st book.
The revenue for selling the 201st book is approximately $R'(200)$:

$$R'(200) = 80 - 0.28(200) = 24.00.$$

Summary: The revenue for selling the 201st book is approximately \$24.00.

10.7.4 A company manufactures and sells x wigits per week. The weekly price-demand function is:

$$p(x) = 20 - x.$$

(a) Find the marginal revenue function.

(b) Sum Use marginal revenue to estimate the additional revenue earned by producing 6 wigits instead of 5 wigits.

10.7.5 The price-demand equation for the sale of Atomic TV sets is

$$p + 0.8x = 500.$$

The price p is in dollars, and x is the demand for Atomic TVs at a price of p dollars.

(a) Find the revenue function $R(x)$ as a function of the demand, x.

(b) Sum Find the marginal revenue at $x = 100$ and write a sentence explaining what this means in terms of TV sales.

10.7.6 The profit function from manufacturing and selling x BabCo Lounge Chairs is

$$P(x) = 40x - 140 - (0.1)x^2.$$

(a) Sum Find the exact additional profit for manufacturing and selling 11 chairs instead of 10 chairs.

(b) Find the marginal profit at $x = 10$.

10.7.7 Acme Office Supplies manufactures file cabinets. The cost (in dollars) of producing x file cabinets is given by

$$C(x) = 1020 + 50x - x^2.$$

(a) Sum Find the exact additional cost of producing 8 file cabinets instead of 7.

(b) Find the marginal cost function.

(c) Sum Use the marginal cost function to approximate the additional cost of producing the 8 file cabinets instead of 7.

10.7.8 Sum The price-demand function for the sale of yo-yos is

$$p = 5.50 - .01x,$$

where p is the price of a yo-yo in dollars, and x is the demand for yo-yos at a price of p dollars. A simple calculation shows that $R'(290) = -.30$. Write a sentence explaining what this means in terms of the yo-yo problem. Be sure to use the correct units for $R'(290)$.

10.7.9 Sum Medtronic Inc. produces insulin pumps. Their profit function is

$$P(x) = 2200x - 0.1x^2 - 350,$$

where $P(x)$ is measured in dollars and x is the number of pumps produced and sold.

Compute Medronic's marginal profit if they produce and sell 1000 pumps.

10.7.10 The profit (in dollars) from producing and selling x garbage disposals is

$$P(x) = -x^2 + 70x.$$

(a) Sum Compute marginal profit at an output level of 10 disposals and write a summary statement.

(b) Evaluate and simplify $P(x+1) - P(x)$.

Answer:

$P(x+1) - P(x) =$

10.7.11 The revenue function for selling x garage-door openers is

$$R(x) = 220x - 0.1x^2,$$

where revenue is in dollars. Find the marginal revenue for $x = 100$ garage-door openers. (Include the correct units of measure in your answer.)

Marginal Revenue:

10.7.12 TweetCo is a boutique store that sells bird-bath fountains. The revenue for selling x bird-bath fountains is $R(x)$ dollars where

$$R(x) = x(200 - 5x).$$

(a) Find the marginal revenue function.

Marginal revenue function:

(b) Use the marginal revenue function to estimate the additional revenue obtained from selling 11 fountains instead of 10.

Estimate of additional revenue:

(c) Find the exact additional revenue obtained from selling 11 fountains instead of 10.

Exact additional revenue:

Section 11.34: Product, Quotient, Generalized Power Rules

11.34.1 Find the derivatives of the following functions and simplify.

(a) $s(x) = (3x - 5x^3)^{1/2} + 100$

(b) $r(x) = \dfrac{5x - 6}{3x + 4}$

Answer only:
$s'(x) = \frac{1}{2}(3x - 5x^3)^{-1/2}(3 - 15x^2)$ and $r'(x) = \frac{38}{(3x+4)^2}$

11.34.2 Let $f(x) = (-x^2 + x + 1)^4$.

(a) Find the derivative $f'(x)$.

(b) Find $f'(1)$.

11.34.3 Let $f(x) = -x^2 + 12x + 1$.

(a) Find the derivative $f'(x)$.

(b) Find $f'(-1)$.

11.34.4 Bling & Co. market faux-diamond studded coffee cups to the local market in Pasadena. Suppose that the number of coffee cups that people are willing to buy per week at a price of p dollars per cup is given by the equation

$$f(p) = \frac{p + 1}{p^2 + 2p + 2}$$

(a) Find $f'(p)$. Simplify your answer.
Complete Solution:

$$\begin{aligned} f'(p) &= \frac{(1)(p^2 + 2p + 2) - (2p + 2)(p + 1)}{(p^2 + 2p + 2)^2} \\ &= \frac{p^2 + 2p + 2 - (2p^2 + 4p + 2)}{(p^2 + 2p + 2)^2} \\ &= \frac{-p^2 - 2p}{(p^2 + 2p + 2)^2} \end{aligned}$$

(b) Is weekly demand increasing, decreasing or neither at a price of \$1 per coffee cup? Why?

$$f'(1) = \frac{-(1)^2 - 2(1)}{((1)^2 + 2(1) + 2)^2}$$

which is clearly negative.

Summary: Since the derivative is negative at a price of \$1 per cup we see that demand is decreasing.

11.34.5 Find the derivatives of the following functions and simplify.

(a) $f(x) = -(x-2)^2 + 3$

(b) $s(x) = 3x^2 + 5x + 100$

(c) $r(x) = \dfrac{3x-2}{(2x+5)^2}$

Complete Solution:

$$\begin{aligned} f'(x) &= -2(x-2) \\ s'(x) &= 6x+5 \\ r'(x) &= \frac{3(2x+5)^2 - 2(2x-5)^1(2)(3x-2)}{(2x-5)^4} \\ &= \frac{3(2x+5) - 4(3x-2)}{(2x-5)^3} \\ &= \frac{6x+15-12x+8}{(2x-5)^3} \\ &= \frac{-6x+23}{(2x-5)^3}. \end{aligned}$$

11.34.6 Find the derivative of the function

$$f(x) = (x^2 + 3x + 1)(14 - 3x^2).$$

11.34.7 Let $f(x) = (x^2 - x + 1)^3$.

a. Find the derivative $f'(x)$.

b. Find $f'(1)$.

11.34.8 Find the derivative of the function

$$f(x) = (x^3 + 4x + 1)(150 - 3x).$$

11.34.9 Find the derivative of the function

$$f(x) = \sqrt{5x+3}.$$

11.34.10 Find the derivative of the function

$$f(x) = \sqrt{x} - \frac{1}{x^3}.$$

11.34.11 Find the derivative of the function

$$f(x) = \frac{2x-1}{3x+5}.$$

11.34.12 $p(x)$ and $R(x)$ stand for price and revenue.

(a) If $R(x) = xp(x)$, what is $R'(x)$ in terms of $p(x)$ and $p'(x)$?

(b) If $P(x) = R(x) - C(x)$, express $P'(x)$ in terms of $R'(x)$, and $C'(x)$

Section 11.7: Elasticity

11.7.1 A company manufactures and sells x clocks per week with weekly demand function: $f(p) = 20 - 2p$ where p is the price per clock.

(a) Compute the elasticity of demand function for this demand function.
Complete Solution:

$$\begin{aligned} E(p) &= \frac{-pf'(p)}{f(p)} \\ &= \frac{2p}{20-2p}. \end{aligned}$$

(b) At $p = \$8$: a price increase of 10% will create a demand decrease of what percent? **Complete Solution:**
Elasticity at $p = 8$ is $E(8) = 4$. Thus the relative rate of decrease in demand is approximately 4 times the relative rate of increase in price.

Summary: Demand is will decrease 40%.

11.7.2 The demand equation p is given by

$$x + p = 4800.$$

(a) Write demand as a function of price.

(b) Find the elasticity of demand at a price of \$800?

(c) Sum If the price increases 10% from a price of \$800, what is the approximate (percentage) change in demand? State whether demand will increase or decrease.

11.7.3 The demand function at a price p is given by

$$f(p) = 3000 - 2p.$$

(a) Find the elasticity of demand.

(b) Sum Is the elasticity of demand at a price of 600 elastic, inelastic, or unitary? Explain.

11.7.4 Me-Tube manufactures bodysuits. The demand function for their suits is

$$x = f(p) = 800 - 2p.$$

The demand $f(p)$ is the number of suits that can be sold at a price of p dollars.

(a) Compute the elasticity of demand for this demand function.

(b) At $p = \$150$ demand is: (circle one)

i. inelastic,
ii. unit
iii. elastic

11.7.5 The elasticity of demand for a wigit is

$$E(p) = \frac{2p}{500 - 2p},$$

where p is the price (in dollars) of one wigit.

(a) At what price is the elasticity of demand unitary (unit).

Answer with units:

(b) Sum If the price increases by 3% from \$100 per wigit, what is the approximate percentage change in demand, and what is the new price?

11.7.6 The neighborhood coffee store sells premium free-trade coffee in one-pound bags. The price-demand equation for this coffee is

$$10p + 0.5x = 100,$$

where x is the number of bags that can be sold at a price of p dollars.

(a) Write the demand as a function, $f(p)$, of price.

$f(p) =$

(b) Compute the elasticity of demand for this demand function.

$E(p) =$

(c) Sum The elasticity of demand at a price of \$6.00 is 1.5. If the price increases by 2%, how will the demand change? (Your summary should state whether demand increases or decreases and by what percentage.)

11.7.7 Icelandic Lavaflow Inc. sells x Joocie smoothies per day with daily demand function: $x = f(p) = 750 - 75p$ where p is the price per smoothie in dollars.

(a) Compute the elasticity of demand function for this demand function.

Answer only:
$E(p) =$

(b) Sum Joocie smoothies are currently selling for \$2.50 each. If we increase the price by 6%, what will be the corresponding percentage change in demand? (Indicate whether this is an increase or a decrease.)

Section 12.5: Absolute Maximums and Minimums

12.5.1 Let $f(x) = x^3 - 27x$. If they exist, find the absolute maximum and the absolute minimum of $f(x)$ on the intervals below. Give explanations and if an absolute max or min does not exist, then say why.

(a) $(-\infty, \infty)$
(b) $[-3, \infty)$
(c) $[-3, 0]$
(d) $[0, 10]$

12.5.2 Let $f(x) = x^2(x^2 - 2) = x^4 - 2x^2$. If they exist, find the absolute max and min of $f(x)$ on the intervals below. Give explanations and if an absolute max or min does not exist, then say why.

(a) $(-\infty, \infty)$
(b) $[-1, 1]$
(c) $[0, \sqrt{2}]$
(d) $[0, 2]$
(e) $(-\infty, 5]$
(f) $[-3, 3]$

12.5.3 Let $f(x) = (x)(x^2 - 12) = x^3 - 12x$. If they exist, find the absolute max and min of $f(x)$ on the intervals below. Give explanations and if an absolute max or min does not exist, then say why.

(a) $(-\infty, \infty)$
(b) $[-\sqrt{12}, \sqrt{12}]$
(c) $[0, 2]$
(d) $[-2, 2]$
(e) $[-2, \infty)$

Section 12.6: Optimization

12.6.1 AmeriCam manufactures and sells motion picture cameras. The price demand equation is

$$p = 2800 - 25x,$$

where p is the price (in dollars) at which x cameras can be sold.

(a) What is the *demand* if the price is \$1000 ?
Complete Solution:
If $p = 1000$, then x must satisfy the equation

$$1000 = 2800 - 25x.$$

Thus

$$x = \frac{2800 - 1000}{25} = 72.$$

Summary: At a price of \$1000 each, the demand is 72 cameras.

(b) The cost to produce x cameras is given by

$$C(x) = 14500 + 800x.$$

and the revenue function is

$$R(x) = x(2800 - 25x).$$

How many cameras should be manufactured and sold to maximize profit?

Complete Solution:
Method 1: Profit is revenue minus cost so

$$P(x) = 2800x - 25x^2 - 14500 + 800x = -25x^2 + 2000x - 14500.$$

The graph of $y = P(x)$ is is a downward facing parabola as the coefficient of the x^2 term is negative. Thus there is a max at the vertex. To find the vertex set $P'(x) = 0$ and solve for x. Well

$$P'(x) = -50x + 2000$$

so,

$$\begin{aligned} P'(x) &= 0 \\ -50x + 2000 &= 0 \\ -50x &= -2000 \\ x &= 40. \end{aligned}$$

Summary: To maximize profit, AmeriCam should manufacture and sell 40 cameras.

OR Method 2: Profit is maximized at the production level, x, where marginal revenue equals marginal cost.

$$\begin{aligned} C'(x) &= 800 \\ R'(x) &= 2800 - 50x. \end{aligned}$$

So we must solve $C'(x) = R'(x)$ for x:

$$\begin{aligned} C'(x) &= R'(x) \\ 800 &= 2800 - 50x \\ 50x &= 2000 \\ x &= 40. \end{aligned}$$

Summary: To maximize profit, AmeriCam should manufacture and sell 40 cameras.

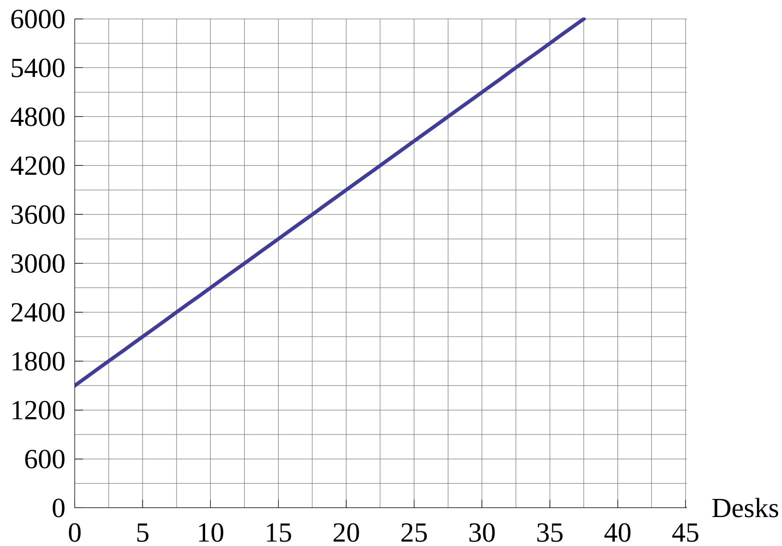
(c) What price should AmeriCam charge for each camera to maximize profit?
Complete Solution:
The price-demand equation is $p = 2800 - 25x$. If demand $x = 400$, then $p = 2800 - 25(40) = 1800$.

Summary: AmeriCam should charge \$1800 per camera.

12.6.2 Seduto Inc. makes combination desk-chairs for classrooms. The weekly revenue and cost functions are:

$$\begin{aligned} R(x) &= 480x - 12x^2 \\ C(x) &= 1500 + 120x, \end{aligned}$$

where revenue and cost are measured in dollars and x is the number of desk-chairs produced and sold. The graph of the cost function is shown below:



(a) The graph of the revenue function $R(x)$ is a parabola. Graph the parabola on the same graph.

(b) Label the x-intercepts and vertex with their coordinates.

(c) Draw a bold line to represent the maximum **profit** graphically.

12.6.3 Sum The cost and revenue functions (in dollars) for producing and selling x Kudsu Sushi machines are given by:

$$C(x) = 40 + 5x, \qquad R(x) = -x^2 + 105x.$$

Find the production level that maximizes profit. Explain your work and give a summary of the cost, revenue, and profit attained at the production level that maximizes profit.

12.6.4 Sum The cost and revenue functions (in dollars) for producing and selling x Ratmeister hamster cages are given by:

$$C(x) = 600 + 4x, \qquad R(x) = -x^2 + 64x.$$

Find the production level that maximizes profit. Explain your work and give a summary of the cost, revenue, and profit attained at the production level that maximizes profit.

12.6.5 Expro Inc. manufactures electronic whiteboards. The cost in dollars to produce x whiteboards is given by

$$C(x) = 300 + 4x.$$

and the revenue function is

$$R(x) = 20x - 0.1x^2.$$

(a) Find the profit function $P(x)$.
Answer only:
$P(x) = -0.1x^2 + 16x - 300$

(b) Sum How many whiteboards should be manufactured and sold to maximize profit?
Answer only:
$x = 80$.

Summary: Expro should sell 80 whiteboards to maximize profit.

(c) If the government imposes a tax of $1.00 per whiteboard, this will effect our production costs and thus our profits. As compared to the situation of the old costs, the output level that would maximize profits using the new costs would be

i. higher.
ii. **lower**.
iii. remain the same.

12.6.6 Retro Phone Inc. manufactures and sells electronic rotary phones. The fixed cost is $200 and the variable cost is $4, so the cost in dollars to produce x phones is given by

$$C(x) = 200 + 4x.$$

The revenue function is

$$R(x) = 22x - 0.3x^2.$$

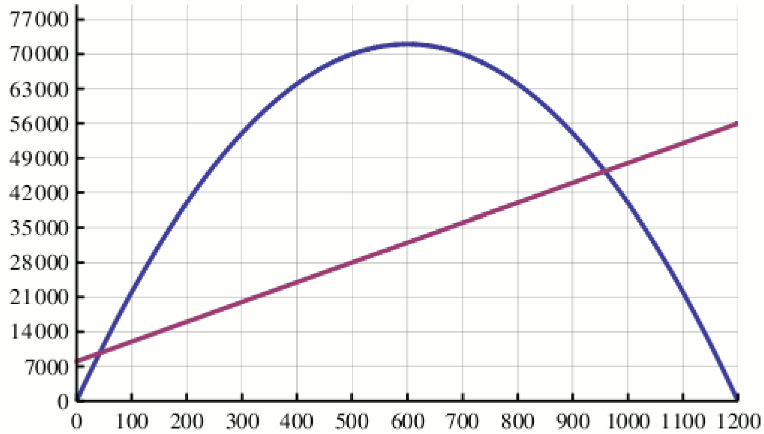
(a) Find the profit function $P(x)$.

(b) Sum How many phones should be produced and sold to maximize profit?

(c) Suppose that the fixed cost to produce phones increases to $300. Using the new fixed cost, how would the number of phones that maximize profit change? The number of phones that maximize profit would (circle one)

i. be higher,
ii. be lower,
iii. remain the same.

12.6.7 The graph of the revenue and cost functions for the production and sale of x units are show below. The cost function is the straight line and the revenue function is the curve.



(a) Use the graph to estimate, to the nearest hundred, the production/sales level that maximizes profit.

Answer with units:

(b) Use the graph to estimate, to the nearest hundred, maximum profit and draw a bold line to represent the maximum profit graphically.

Answer with units:

12.6.8 A company manufactures and sells halloween candy each year. Output x is measured in tons of candy. The revenue and cost functions are:

$$R(x) = 18x - x^2, \qquad C(x) = 25 + 4x.$$

(a) Find the production/sales level that maximizes profit.

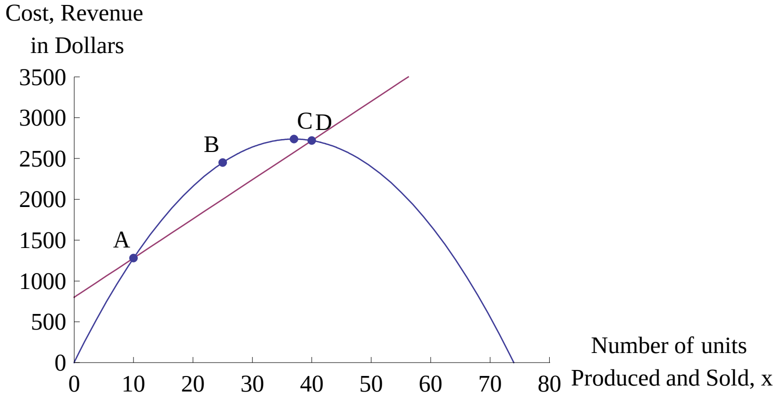
Answer with units:

(b) Sum If the fixed costs to produce candy is raised from \$25 to \$30 how will your answer to part (a) change? Why?

12.6.9 The cost $C(x)$ and revenue $R(x)$ for producing and selling x units are:

$$C(x) = 800 + 48x, \qquad R(x) = x(148 - 2x) = 148x - 2x^2.$$

Both cost and revenue are in dollars. Their graphs are shown below.



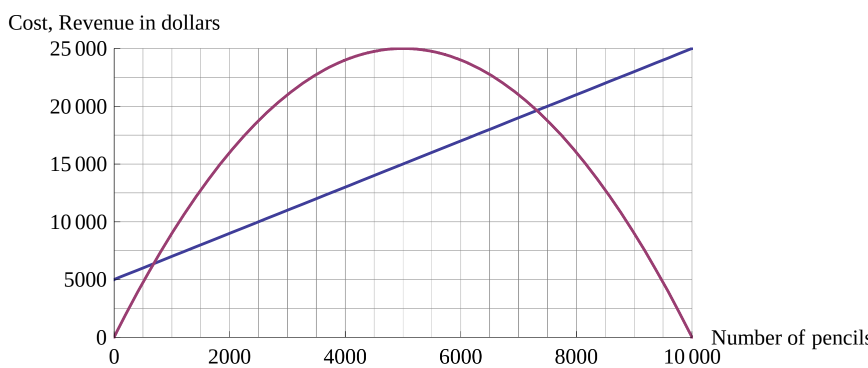
a. Which of the four points on the graph shows the revenue for the level x where **profit** is maximum?

Circle one: Point A Point B Point C Point D

b. Sum Find the value of x at which the marginal cost is equal to the marginal revenue.

c. Sum How many units should be produced and sold for maximum profit, and what is the maximum profit?

12.6.10 An office supply company manufactures and sells mechanical pencils. The graphs in the picture below represent the annual cost function $C(x)$ (the line) and the annual revenue function $R(x)$ (the parabola) for the manufacturing and sale of x pencils. Both cost and revenue are in dollars.



Use the graph to answer the following questions:

(a) Estimate the maximum **revenue** and the manufacturing/sales level that realizes the maximum revenue.

Maximum Revenue with units:

Manufacturing/sales level (with units) that realizes the maximum revenue:

(b) Mark the vertical line segment on the graph that represents the maximum **profit**.

(c) The company is considering restructuring its assembly plant that will reduce annual fixed costs by \$500. How will the restructuring change the manufacturing/sales level that maximizes profit?

☐ Level does not change ☐ Level Increases ☐ Level decreases

12.6.11 The marketing department at Wellington Tires has found that the revenue (in dollars) earned when x tires are sold is given by

$$R(x) = -x^2 + 71x.$$

Suppose that the cost equation is $C(x) = 31x + 250$

(a) Find profit $P(x)$ as function of x.
$P(x) =$

(b) What is the number of tires that must be produced and sold in order to maximize profit?
Answer with units:

(c) What is the maximum profit?
Answer with units:

Section 4.123: Systems of Linear Equations, Augmented Matrix, Elimination

4.123.1 Solve this system of linear equations:

$$\begin{aligned} 4x + 3y &= 37 \\ -3x + 2y &= -32. \end{aligned}$$

4.123.2 Solve this system of linear equations:

$$\begin{aligned} 4x + 3y &= -30 \\ -8x - 6y &= 0. \end{aligned}$$

4.123.3 Find the coordinates (x, y) of the point of intersection for the lines with the equations:

$$\begin{aligned} x + 2y &= -4 \\ 3x + 4y &= -2 \end{aligned}$$

4.123.4 Find the coordinates (x, y) of the point of intersection for the lines with the equations:

$$\begin{aligned} x + 2y &= 7 \\ 3x + 6y &= 21 \end{aligned}$$

4.123.5 The supply and demand equations for a product are given below:

$$\begin{aligned} \text{supply} \quad q - 3p &= -1 \\ \text{demand} \quad q + 2p &= 4. \end{aligned}$$

(a) Find an augmented matrix that corresponds to this system of equations.
Complete Solution:
The augmented matrix for the system is

$$A = \begin{bmatrix} 1 & -3 & -1 \\ 1 & 2 & 4 \end{bmatrix}$$

(b) Put the matrix from part (a) in row reduced echelon form.
Answer only:
The row reduced echelon form is

$$R = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix}.$$

(Student should show reduction steps.)

(c) How many solutions does the system have?
Complete Solution:
There is only one solution.

4.123.6 The supply and demand equations for a product are given below:

$$\begin{array}{lrcr} \text{supply} & 10q - 25p & = & -50 \\ \text{demand} & q + p & = & 30. \end{array}$$

(a) Find an augmented matrix that corresponds to this system of equations.
Complete Solution:

$$A = \begin{bmatrix} 10 & -25 & -50 \\ 1 & 1 & 30 \end{bmatrix}$$

(b) Put the matrix from part (a) in row reduced echelon form.
Complete Solution:

$$\begin{array}{rl}
\begin{bmatrix} 10 & -25 & -50 \\ 1 & 1 & 30 \end{bmatrix} & R_1 \to \frac{1}{5} R_1 \\
\begin{bmatrix} 2 & -5 & -10 \\ 1 & 1 & 30 \end{bmatrix} & R_2 \to 2R_2 - R_1 \\
\begin{bmatrix} 2 & -5 & -10 \\ 0 & 7 & 70 \end{bmatrix} & R_2 \to \frac{1}{7} R_2 \\
\begin{bmatrix} 2 & -5 & -10 \\ 0 & 1 & 10 \end{bmatrix} & R_1 \to R_1 + 5R_2 \\
\begin{bmatrix} 2 & 0 & 40 \\ 0 & 1 & 10 \end{bmatrix} & R_1 \to \frac{1}{2} R_1 \\
\begin{bmatrix} 1 & 0 & 20 \\ 0 & 1 & 10 \end{bmatrix} & \text{Row-reduced form}
\end{array}$$

(c) How many solutions does the system have?
Complete Solution:
The system has one unique solution.

4.123.7 The supply and demand equations for a product are given below:

$$\begin{array}{lrcr} \text{supply} & q - 3p & = & -5 \\ \text{demand} & 5q + 2p & = & 60. \end{array}$$

(a) Find an augmented matrix that corresponds to this system of equations.

(b) Put the matrix from part (a) in row reduced echelon form.

(c) How many solutions does the system have?

4.123.8 The following system of linear equations has a unique solution.

$$\begin{aligned} x - 2y + 3z &= 3 \\ -x + 4y + z &= 5 \\ -x + 2y - 2z &= -2 \end{aligned}$$

(a) Write the augmented matrix for this system of equations.

(b) Find the solution of this system of linear equations. You must use Gauss-Jordan elimination, doing each step separately.

4.123.9 The supply and demand equations for a product are given below:

$$\begin{array}{lrcl} \text{supply} & 3q - 8p & = & -4 \\ \text{demand} & q + p & = & 28. \end{array}$$

(a) Find an augmented matrix that corresponds to this system of equations.

(b) Put the matrix from part (a) in row reduced echelon form.

(c) How many solutions does the system have?

4.123.10 Write the augmented matrix for the following system of linear equations:

$$\begin{array}{rcrcrl} 2x & + & y & - & z & = 9 \\ -x & - & 4y & + & 2z & = 0 \end{array}$$

Augmented matrix:

4.123.11 Find the point of equilibrium for the following supply and demand equations; x is the number of units **in hundreds** and p is the price per unit (in dollars). (Equilibrium is where supply equals demand.)

$$\begin{array}{lcl} \text{supply} & p & = 0.3x + 3 \\ \text{demand} & p & = -1.7x + 15 \end{array} .$$

Number of units:

Price per unit:

4.123.12 Consider the following augmented matrix.

$$A = \left[\begin{array}{ccc|c} 2 & 2 & -1 & 1 \\ 4 & 6 & -4 & 10 \\ 7 & 1 & 4 & -4 \end{array} \right].$$

(a) Find the row reduced echelon form of the matrix A.

(b) Write down the system of equations that the matrix A corresponds to.

(c) How many solutions does it have?

(d) Find all solutions.

4.123.13 Consider the following augmented matrix.

$$A = \left[\begin{array}{ccc|c} 3 & 9 & 2 & 1 \\ 6 & 18 & -4 & 10 \\ 1 & 3 & 5 & -4 \end{array} \right].$$

(a) Find the row reduced echelon form of the matrix A.

(b) Write down the system of equations that the matrix A corresponds to.

(c) How many solutions does it have?

(d) Find all solutions.

Section 4.4: Matrix Operations, Matrix Models

4.4.1 Both of the Mathematics Departments at CSU Northridge and Fullerton give final exams in College Algebra (CA) and the Mathematical Methods for Business (BM). This uses resources from the department faculty (F) to make the exams, the staff (S) to copy the exams and the teaching assistants (T) to proctor the exams. Here are the labor-hour and wage requirements for administering each exam:

	Faculty	Staff	Teaching Assistants
Business Math Exam	5.0 hrs work	0.5 hrs work	2.0 hrs work
College Algebra Exam	7.0 hrs work	1.0 hrs work	2.0 hrs work

	CSUN	CSU, Fullerton
Faculty	\$40 per hour	\$50 per hour
Staff	\$14 per hour	\$16 per hour
Teaching Assistants	\$8 per hour	\$10 per hour

The labor-hours and wage information is given in the following matrices:

$$M = \begin{bmatrix} 5.0 & 0.5 & 2.0 \\ 7.0 & 1.0 & 2.0 \end{bmatrix}, \qquad N = \begin{bmatrix} 40 & 50 \\ 14 & 16 \\ 8 & 10 \end{bmatrix}$$

(a) Compute the product MN
Answer only:

$$MN = \begin{bmatrix} 223 & 278 \\ 310 & 386 \end{bmatrix}$$

(b) What is the $(1, 2)$-entry (also known as R1C2) of matrix MN and what does it mean?

Summary: The $(1, 2)$-entry of MN is 278. \$278 are spent on labor to make up the Business Math Exam at CSU Fullerton.

4.4.2 Delta Duplex Properties builds two-family dwellings. They have two models: Economy Model, Deluxe Model. The cost to build depends on the square footage of the building and the size of the lot. Of course, the Deluxe Model building and lot are larger than the Economy Model. Square footage and costs per square foot are given in the tables below:

	Size of building	Size of lot
Economy Model	2300	7000
Deluxe Model	3000	9000

Sizes are given in square feet.

Building cost	Lot cost
\$300	\$100

Costs are given in dollars per square foot.

The size and cost information is given in the following matrices:

$$S = \begin{bmatrix} 2300 & 7000 \\ 3000 & 9000 \end{bmatrix}, \qquad C = \begin{bmatrix} 300 \\ 100 \end{bmatrix}.$$

(a) Compute the product SC.

(b) Sum Explain what each of the entries in the product SC means.

4.4.3 There are two food stores near Mrs. Garcia's house, Hons (H) and Trader Vo (TV). She needs to buy 8 apples, 5 bananas, and 2 bunches of cilantro. The prices for each of these items in the two stores are given in the table below:

	Apples	Bananas	Cilantro
H	\$0.40 each	\$0.50 each	\$1.20 each
TV	\$0.55 each	\$0.35 each	\$1.00 each

Mrs. Garcia's shopping list is given in the next table:

Apples	Bananas	Cilantro
8	5	2

The prices and shopping list are given in the following matrices:

$$P = \begin{bmatrix} 0.40 & 0.50 & 1.20 \\ 0.55 & 0.35 & 1.00 \end{bmatrix}, \qquad S = \begin{bmatrix} 8 \\ 5 \\ 2 \end{bmatrix}$$

(a) Compute the product PS

(b) What is the $(2,1)$-entry (also known as R2C1) of matrix PS and what does it mean?

4.4.4 CarCoCo (CCC) and AceAuto (AA) are competing auto body shops that specialize in painting cars. Three types of labor are required to complete a paint job: Sanding/Filling, Masking, and Spraying. The number of hours required to complete each job at the two shops are given in the first table and the matrix L. Labor costs, in dollars per hour, are given in the second table and the matrix C.

	Sanding/ Filling	Masking	Spraying
CCC	6	8	2
AA	5	4	2

Sanding/Filling	\$16.00
Masking	\$10.00
Spraying	\$25.00

$$L = \begin{bmatrix} 6 & 8 & 2 \\ 5 & 4 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} 16.00 \\ 10.00 \\ 25.00 \end{bmatrix}.$$

A) Compute the matrix product LC

B) What is the (2,1) entry of LC and what does it mean?

4.4.5 (a) Put the matrix A in row reduced echelon form. Describe each row operation that is used.

$$A = \begin{bmatrix} 1 & 3 & 0 & 5 \\ 4 & 12 & 2 & 0 \end{bmatrix}$$

Answer only:

$$\begin{bmatrix} 1 & 3 & 0 & 5 \\ 0 & 0 & 1 & -10 \end{bmatrix}$$

(b) Compute the following:

$$\begin{bmatrix} 1 & 3 & 2 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 2 \\ 4 & 0 \end{bmatrix} - 3 \begin{bmatrix} 0 & 3 \\ 1 & 0 \end{bmatrix}$$

Answer only:

$$\begin{bmatrix} 9 & -1 \\ 18 & 2 \end{bmatrix}$$

4.4.6 Dorothy's Costume Company produces three costumes: Tinman, Scarecrow, and Lion. They sell for \$20, \$10, and \$30, respectively. Dorothy has two stores, one in Kansas and one in Oz. Last week, the store in Kansas sold 4 Tinman costumes, 6 Scarecrow costumes, and 1 Lion costume. The store in Oz sold 3 Tinman costumes, no Scarecrow costumes, and 2 Lion costumes.

(a) Record the price information in a 1×3 matrix, M.

(b) Record the sales information in a 3×2 matrix, N.

(c) Find the product of the two matrices, MN.

(d) Sum What does the entry in the 1st row and 2nd column of matrix MN mean to Dorothy?

4.4.7 A computer company has production facilities in two different countries; country A and country B. The labor-hour and wage requirements for the manufacture of laptop and desktop models are as follows:

Labor Hours per Computer:

	Assembly	Testing	Packing & Shipping
Laptop	3 hours	1.5 hours	.50 hours
Desktop	4 hours	2.5 hours	.75 hours

Hourly Wages

	Country A	Country B
Assembly	\$14 per hour	\$13 per hour
Testing	\$16 per hour	\$20 per hour
Packing and Shipping	\$12 per hour	\$10 per hour

The labor hours and wages are given in the following matrices:

$$L = \begin{bmatrix} 3 & 1.5 & .50 \\ 4 & 2.5 & .75 \end{bmatrix}, \qquad W = \begin{bmatrix} 14 & 13 \\ 16 & 20 \\ 12 & 10 \end{bmatrix}.$$

(a) Compute the product LW

(b) Sum What is the $(2,1)$-entry (also known as R2C1) of matrix LW and what does it mean?

4.4.8 For the matrices A and B below, fill in the missing entry of the product AB. (Show your work.)

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 2 & 3 \\ 1 & 1 \\ -2 & 1 \end{bmatrix}, \qquad AB = \begin{bmatrix} 4 & \square \\ -1 & 2 \end{bmatrix}$$

4.4.9 A patio furniture company makes two models of chairs, the Econo-line and the Deluxe. Each model requires two materials, aluminum and plastic webbing. The amount of each material required to make each model is given in the table below. The amount of aluminum is in pounds and webbing is in yards.

	Aluminum	Webbing
Econo-line	1.2	3.0
Deluxe	2.2	5.0

The cost of one pound of aluminum is \$4.20 and the cost of one yard of webbing is \$2.50.

The materials matrix M, the cost matrix C, and their prouduct MC are given below:

$$M = \begin{bmatrix} 1.2 & 3.0 \\ 2.2 & 5.0 \end{bmatrix}, \qquad C = \begin{bmatrix} 4.20 \\ 2.50 \end{bmatrix}, \qquad MC = \begin{bmatrix} 12.54 \\ 21.74 \end{bmatrix}.$$

Which of the following four statements correctly describes the meaning of the entry 21.74 in the matrix product MC?

Circle one:

A The total cost to make one Econo-line chair is \$21.74
B The total cost to make one Deluxe chair is \$21.74
C The cost of aluminum to make one Econo-line chair and one Deluxe chair is \$21.74.
D The cost of webbing to make one Econo-line chair and one Deluxe chair is \$21.74.

4.4.10 Anna and Ben take classes at a community college CACC and a local university CSUM. The number of credit hours taken and the costs per credit hour are as follows:

	CACC	CSUM
Anna	6 credit hours	5 credit hours
Ben	3 credit hours	10 credit hours

	Cost of tuition per credit hour	Other costs per credit hour
CACC	\$ 50.00	\$ 3.00
CSUM	\$ 100.00	\$ 5.00

(a) Write the credit hours information in a matrix M.
Write the cost information in a matrix N.

(b) Compute the product MN.

(c) What is the (2,1)-entry of matrix MN and what does it mean?