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ON FIBER-PRESERVING ISOTOPIES OF SURFACE HOMEOMORPHISMS

TERRY FULLER

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ABSTRACT. We show that there are homeomorphisms of closed oriented genus g surfaces Σ_g which are fiber-preserving with respect to an irregular branched covering $\Sigma_g \to S^2$ and isotopic to the identity, but which are not fiber-isotopic to the identity.

0. Introduction

The study of branched coverings of surfaces is a classical subject which continues to play a role in modern topology, often in application to the study of manifolds of higher dimension. For instance, branched coverings of surfaces play a key role in the famous result of Hilden [5] and Montesinos [8] that every closed orientable 3-manifold is a 3-fold branched cover of S^3. More recently, branched coverings of the 2-sphere have been used to study symplectic 4-dimensional manifolds in [3] and [10]. In particular, the behavior of surface homeomorphisms with respect to branched coverings is used in all of these applications.

Let $\pi : \Sigma_g \to S^2$ denote a branched covering of the 2-sphere by a closed oriented genus g surface Σ_g. A homeomorphism $h : \Sigma_g \to \Sigma_g$ is called *fiber-preserving* (with respect to π) if $\pi(x) = \pi(y)$ implies $\pi(h(x)) = \pi(h(y))$. If h is fiber-preserving and isotopic to the identity, then we say that h is *fiber-isotopic* to the identity if we find an isotopy h_t between h and the identity with h_t fiber-preserving for each $0 \leq t \leq 1$. In [2], Birman and Hilden prove the following theorem.

Theorem 1 (Birman, Hilden). *Let $\pi : \Sigma_g \to S^2, g \geq 2,$ denote a regular branched covering with a finite group of covering translations which fix each branch point. Let $h : \Sigma_g \to \Sigma_g$ be a fiber-preserving homeomorphism which is isotopic to the identity. Then h is fiber-isotopic to the identity.*

In this paper we show that the hypothesis that π is regular is indeed necessary, by constructing explicit fiber-preserving homeomorphisms which are isotopic to the identity, but which are not fiber-isotopic to the identity with respect to an irregular branched covering.

The method of proof is a reversal of the usual direction of application, since we use the topology of symplectic Lefschetz fibrations on 4-manifolds to find homeomorphisms of surfaces that are not fiber-isotopic to the identity. The topology of a

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Lefschetz fibration over the 2-sphere is traditionally understood through its global monodromy, which is a homeomorphism of the fiber surface isotopic to the identity. When the global monodromy of a Lefschetz fibration $M^4 \to S^2$ represents an element of the hyperelliptic mapping class group of the fiber, it has been established independently by the author [3] and Siebert and Tian [10] that M may be obtained as a 2-fold branched cover of a (possibly blown up) S^2-bundle over S^2, branched over an embedded surface. The proof given in [3] does this by considering 2-fold branched covers $\pi : \Sigma_g \to S^2$, and makes essential use of Theorem 1. In an attempt to realize nonhyperelliptic Lefschetz fibrations as branched covers, it is tempting to try and generalize the arguments of [3] by considering instead irregular simple 3-fold covers $\pi : \Sigma_g \to S^2$. We show in Section 1 that doing this allows one to describe the complement of a neighborhood of a nonsingular fiber F as a simple 3-fold cover. The existence of nonfiber-isotopic homeomorphisms then follows in Section 2 by considering the global monodromies of these Lefschetz fibrations, and stems from the observation that if an analog of Theorem 1 held for irregular coverings, then we could extend the branched covering constructed in Section 1 over F, thereby realizing M^4 as a 3-fold covering in a way that is forbidden by the results of Siebert and Tian [10].

Notation. For a simple closed curve β on an oriented surface, D_β denotes a righthanded Dehn twist about β. For an arc δ on an oriented surface, D_δ denotes a righthanded "disk twist" about δ; this is the homeomorphism which rotates a small disk neighborhood of δ by 180°, interchanging the endpoints of δ (see [1]).

1. Three-fold branched covering spaces and Lefschetz fibrations

We begin with a useful criterion for recognizing fiber-preserving homeomorphisms.

Lemma 2. *A homeomorphism $h : \Sigma_g \to \Sigma_g$ is fiber-preserving with respect to a branched covering $\pi : \Sigma_g \to S^2$ if and only if there exists a homeomorphism $k : S^2 \to S^2$ such that the diagram*

$$\begin{array}{ccc} \Sigma_g & \xrightarrow{h} & \Sigma_g \\ {\scriptstyle\pi}\downarrow & & \downarrow{\scriptstyle\pi} \\ S^2 & \xrightarrow{k} & S^2 \end{array}$$

commutes.

The proof of Lemma 2 is an easy exercise in checking definitions, and we omit it.

For the remainder of this paper, let $\pi : \Sigma_g \to S^2$ denote the unique (up to equivalence) irregular simple 3-fold branched covering. We refer the reader to [1] for a general introduction to branched coverings of surfaces. The branch set $B = \{x_1, \ldots, x_{2g+4}\}$ necessarily consists of $2g+4$ points. We assume that the branched covering is normalized according to a Hurwitz system of arcs so that the monodromy about the branch points $x_1, \ldots, x_{2g+2}$ is (12), and the monodromy about the branch points x_{2g+3}, x_{2g+4} is (23); thus we write $B = P \cup R$, with $P = \{x_1, \ldots, x_{2g+2}\}$ and $R = \{x_{2g+3}, x_{2g+4}\}$.

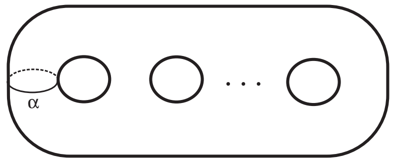

FIGURE 1. Σ_g

Definition. A nonseparating simple closed curve $\gamma \subset \Sigma_g$ is called *symmetric* (with respect to π) if there exists an arc $\delta \subset S^2$ with endpoints in B and otherwise disjoint from B such that

(a) $\pi^{-1}(\delta)$ consists of the disjoint union of γ and an embedded arc γ';
(b) $\pi|\gamma$ is a 2-fold branched cover of δ; and
(c) $\pi|\gamma' : \gamma' \to \delta$ is a homeomorphism.

Lemma 3. *Let $\gamma \subset \Sigma_g$ be an arbitrary nonseparating simple closed curve. Then γ is isotopic to a symmetric curve.*

Proof. There is a homeomorphism h_0 of Σ_g taking the symmetric curve α pictured in Figure 1 to γ.

There exist homeomorphisms $h : \Sigma_g \to \Sigma_g$ and $k : S^2 \to S^2$ with h isotopic to h_0 for which the diagram

$$\begin{tikzcd} \Sigma_g \arrow[r, "h"] \arrow[d, "\pi"'] & \Sigma_g \arrow[d, "\pi"] \\ S^2 \arrow[r, "k"] & S^2 \end{tikzcd}$$

commutes (see [1] or [5]). Setting $\delta = k\pi(\alpha)$ shows that $h(\alpha)$ is a symmetric curve isotopic to γ. □

Remark 4. The existence of the homeomorphisms h and k in the above proof follows from showing that a standard collection of Dehn twist generators for the mapping class group of Σ_g are lifts under π of homeomorphisms of S^2 [1]. Specifically, each Dehn twist generator D_β about a closed curve β is isotopic to the lift of a disk twist $D_{\pi(\beta)}$ about the arc $\pi(\beta)$. It is easy to check that the arcs $\pi(\beta)$ corresponding to each generator of the mapping class group have endpoints either both in P or both in R. As a result, we may assume that k fixes both P and R setwise. In general, the arc δ in the definition of a symmetric curve is not unique, as can be seen for instance by choosing other curves to play the role of α in the proof of Lemma 3. However, if we do choose δ as in the proof of Lemma 3, it will have both endpoints on P, since $\pi(\alpha)$ has both endpoints on P and k fixes the set P.

Lemma 5 (See [1] or [5]). *Let γ be a nonseparating symmetric simple closed curve, with γ' and δ as in the definition above. Then the diagram*

$$\begin{tikzcd} \Sigma_g \arrow[r, "D_\gamma \circ D_{\gamma'}"] \arrow[d, "\pi"'] & \Sigma_g \arrow[d, "\pi"] \\ S^2 \arrow[r, "D_\delta"] & S^2 \end{tikzcd}$$

commutes.

Notation. We denote the composition $D_\gamma \circ D_{\gamma'}$ by $D_{\gamma,\gamma'}$. Since γ and γ' are disjoint, the order of composition does not matter. Since $D_{\gamma'}$ is isotopic to the identity, it follows that $D_{\gamma,\gamma'}$ is isotopic to D_γ.

Definitions. Let M be a compact, oriented smooth 4-manifold, and let C be a compact, oriented surface. A proper smooth map $f : M \to C$ is a *Lefschetz fibration* if each critical point has an orientation-preserving complex coordinate chart on which $f : \mathbb{C}^2 \to \mathbb{C}$ is given by $f(z_1, z_2) = z_1^2 + z_2^2$.

A nonsingular fiber of f is diffeomorphic to a smooth genus g surface Σ_g, and we call f a *genus g Lefschetz fibration.* All singular fibers of f have nodal singularities: they are immersed with a positive transverse self-intersection. Each singularity corresponds to an embedded circle–known as a *vanishing cycle*–on a nearby nonsingular fiber, and the singular fiber can be viewed as the result of collapsing the vanishing cycle to a point to create a transverse self-intersection. The global topology of a Lefschetz fibration $f : M \to S^2$ is frequently understood via an equivalence class of homeomorphisms of Σ_g known as the *global monodromy* of f. We can obtain a representative of this equivalence class by deleting a regular neighborhood $\nu(F) \cong \Sigma_g \times D^2$ of a nonsingular fiber F, forming $M_0 = M - \nu(F)$. This gives by restriction a Lefschetz fibration $f_0 : M_0 \to D^2$, with the boundary ∂M_0 a Σ_g-bundle over S^1. The monodromy $h : \Sigma_g \to \Sigma_g$ of this Σ_g-bundle is a representative of the global monodromy of f. (The full equivalence class that is the global monodromy of f comes from taking into account choices involved in the above description.) Since $\partial M_0 = \partial\nu(F) \cong \Sigma_g \times S^1$, h must be isotopic to the identity. A comprehensive introduction to the topology of Lefschetz fibrations may be found in [4].

Remark 6. Gompf has shown that if M^4 admits a Lefschetz fibration as in our definition with $[F] \neq 0$ in $H_2(M^4; \mathbb{R})$, then M^4 is symplectic with symplectic fibers [4]. Therefore some authors now refer to this definition as a *symplectic* Lefschetz fibration. One may study a larger class of fibrations by eliminating the condition that the coordinate charts in our definition are orientation-preserving; these are called *achiral* Lefschetz fibrations. The total space of an achiral Lefschetz fibration is not symplectic, in general.

Theorem 7. *Let $M \to S^2$ be a genus g Lefschetz fibration, and assume that all of the vanishing cycles of this fibration are nonseparating curves. Let $M_0 = M - \nu(F)$ denote the complement of a regular neighborhood of a nonsingular fiber F of this fibration. Then M_0 is a simple 3-fold cover of $S^2 \times D^2$, branched over an embedded surface.*

Proof. As is well known, the existence of a Lefschetz fibration $f_0 : M_0 \to D^2$ allows one to describe M_0 as a handlebody, in a standard way (see [4] or [7]). More precisely, if the Lefschetz fibration $f_0 : M_0 \to D^2$ has μ singular fibers, then we may express the global monodromy of f_0 as $D_{\gamma_\mu} \circ \cdots \circ D_{\gamma_1}$, where γ_i denotes the vanishing cycle corresponding to the ith singular fiber, and D_{γ_i} denotes a righthanded Dehn twist about the curve γ_i. The total space M_0 is diffeomorphic to the handlebody obtained by attaching μ 2-handles $H_1^2, \ldots, H_\mu^2$ to $\Sigma_g \times D^2$, with each H_i^2 attached along the vanishing cycle $\gamma_i \subset \Sigma_g \times \{pt.\} \subset \partial(\Sigma_g \times D^2)$. The handles are attached in the cyclic order given by the factorization of the global monodromy as the composition of Dehn twists, as one traverses the S^1 factor of $\partial(\Sigma_g \times D^2) = \Sigma_g \times S^1$. In addition, each H_i^2 is required to be attached with framing

-1 relative to the framing on γ_i induced from the product structure on $\Sigma_g \times S^1$. We prove the theorem by adapting the techniques in [9] to produce a branched covering on this handlebody.

We begin the construction by noting that $\pi \times id : \Sigma_g \times D^2 \to S^2 \times D^2$ describes $\Sigma_g \times D^2$ as a simple 3-fold branched covering. To extend this branched covering to all of M_0, we apply Lemma 3 to "symmetrize" each vanishing cycle γ_i, obtaining arcs $\delta_i \subset S^2$ and $\gamma_i' \subset \Sigma_g$. A regular neighborhood $\nu(\delta_i)$ in $S^2 \times S^1$ lifts under $\pi \times id$ to the disjoint union of regular neighborhoods $\nu(\gamma_i) \cup \nu(\gamma_i')$ in $\Sigma_g \times S^1$, with $\nu(\gamma_i)$ mapped to $\nu(\delta_i)$ as a 2-fold branched cover, and $\nu(\gamma_i')$ mapped homeomorphically to $\nu(\delta_i)$. We let $h_i : (\partial D^2 \times D^2)_i \to \nu(\gamma_i)$ for $1 \le i \le \mu$ denote the attaching map of H_i^2, and write $g_i : H_i^2 \to H_i^2/V$ for the 2-fold cover induced from the involution $V : D^2 \times D^2 \to D^2 \times D^2$ which is reflection through $D^1 \times D^1$. We may assume, after an isotopy, that the involution $h_i \circ V \circ h_i^{-1}$ on $\nu(\gamma_i)$ agrees with the involution on $\nu(\gamma_i)$ arising from the projection of $\nu(\gamma_i)$ to $\nu(\delta_i)$, so this involution extends over H_i^2. Letting ρ_i denote the homeomorphism from $\nu(\gamma_i')$ to $\nu(\delta_i)$, we can then form the map $(\pi \times id) \bigcup_{i=1}^{\mu} g_i \bigcup_{i=1}^{\mu} id$ from

$$(\Sigma_g \times D^2) \bigcup_{i=1}^{\mu} H_i^2 \bigcup_{i=1}^{\mu} H_i^2/V$$

to

$$(S^2 \times D^2) \bigcup_{i=1}^{\mu} H_i^2/V,$$

where the 4-balls H_i^2/V are attached in the domain by $\rho_i^{-1} \circ (\pi \times id) \circ h_i \circ g_i^{-1}$ and in the range by $(\pi \times id) \circ h_i \circ g_i^{-1}$. This map is a simple 3-fold cover. However, the attachments of H_i^2/V to both the domain and range are merely boundary connected sums with a 4-ball, and do not change the manifolds. Thus we have constructed a simple 3-fold cover $M_0 \to S^2 \times D^2$. □

Remark 8. The branched covering of Theorem 7 restricted to the boundary can be described in more detail. The description $M_0 \cong (\Sigma_g \times D^2) \bigcup_{i=1}^{\mu} H_i^2 \bigcup_{i=1}^{\mu} H_i^2/V$ when restricted to the boundary yields a description of ∂M_0 as the result of performing μ Dehn surgeries on $\Sigma_g \times S^1$, and gives an explicit representation

$$\partial M_0 \cong \frac{\Sigma_g \times I}{(x,1) \sim (h(x),0)}$$

of the boundary as the mapping torus of $h = D_{\gamma_\mu,\gamma_\mu'} \circ \cdots \circ D_{\gamma_1,\gamma_1'}$. (Note that the addition of the 4-balls H_i^2/V in the proof of Theorem 7 has the effect of changing the representation of the global monodromy from $D_{\gamma_\mu} \circ \cdots \circ D_{\gamma_1}$ to $D_{\gamma_\mu,\gamma_\mu'} \circ \cdots \circ D_{\gamma_1,\gamma_1'}$.) Restricted to the boundary, the covering projection in the proof of Theorem 7 can be described in terms of mapping tori by beginning with $\pi \times id : \Sigma_g \times I \to S^2 \times I$, and forming the quotient map

$$\frac{\Sigma_g \times I}{(x,1) \sim (h(x),0)} \longrightarrow \frac{S^2 \times I}{(\pi(x),1) \sim (k \circ \pi(x),0)},$$

where $k = D_{\delta_\mu} \circ \cdots \circ D_{\delta_1}$ denotes the product of disk twists about the corresponding δ_i's. Since $\pi \circ h = k \circ \pi$ by Lemma 5, this map is well defined.

Remark 9. The branch set for the branched covering may be explicitly visualized. The branch set for $\pi \times id$ is the $2g + 4$ disks $(P \cup R) \times D^2$, which can be seen in

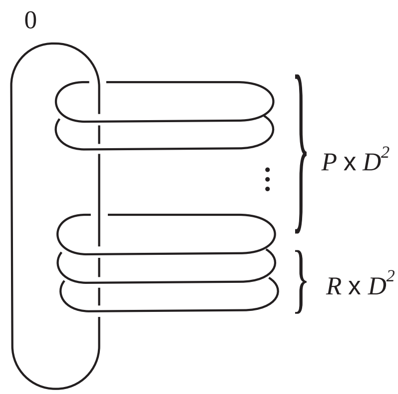

FIGURE 2.

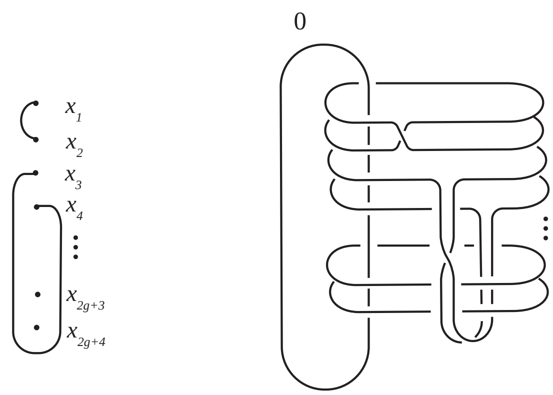

FIGURE 3.

$S^2 \times D^2$ as in Figure 2. The fixed set of the involution V on each 2-handle H_i^2 in the proof of Theorem 7 is a disk, which is mapped under $(\pi \times id) \circ h_i$ to a band attached to $(P \cup R) \times D^2$ with core δ_i. The band will appear with a negative one-half twist, relative to the "product band" $\delta_i \times (pt. - \epsilon, pt. + \epsilon) \subset S^2 \times S^1$. (This half twist arises because the attaching map h_i of H_i^2 has framing -1, relative to the product.) Figure 3 shows how two such bands appear for typical δ_i. Hence the branch set can be described as a "ribbon manifold" of the sort shown in Figure 3, with one band attached along δ_i for each vanishing cycle of the fibration $f_0 : M_0 \to D^2$. Note that because all of the endpoints of the δ_i lie on P (see Remark 4), no bands are ever attached to the lower two disks $R \times D^2$. This gives the following Corollary.

Corollary 10. *The branch surface for the branched covering $M_0 \to S^2 \times D^2$ described in Theorem 7 has at least three connected compenents.*

2. Isotopies of Riemann surfaces

We are ready for our main theorem.

Theorem 11. *Let $\gamma_1, \ldots, \gamma_\mu$ be a collection of nonseparating symmetric simple closed curves on Σ_g. Let $h : \Sigma_g \to \Sigma_g$ be the homeomorphism $h = D_{\gamma_\mu, \gamma'_\mu} \circ \cdots \circ$*

$D_{\gamma_1,\gamma_1'}$. *Then h is fiber-preserving. Assume, in addition, that h is isotopic to the identity. Then h is not fiber-isotopic to the identity with respect to π.*

Proof. The statement that h is fiber-preserving follows immediately from Lemma 2 and Lemma 5. To prove the last statement, let $f_0 : M_0 \to D^2$ denote the Lefschetz fibration whose vanishing cycles are given by the ordered collection of curves $\gamma_1, \dots, \gamma_\mu$. Let h_t for $0 \le t \le 1$ denote an isotopy between $h = h_0$ and the identity, and let

$$W = \frac{\Sigma_g \times I \times I}{(x,1,t) \sim (h_t(x),0,t)}.$$

Each $W_s = \Sigma_g \times I \times I/(x,1,s) \sim (h_t(x),0,s)$ for a fixed $0 \le s \le 1$ is a Σ_g-bundle over S^1, and $W_0 \cong \partial M_0$ by Remark 8. Since h is isotopic to the identity via h_t, this gives a trivial product cobordism between the two components of $\partial W = \partial M_0 \cup (\Sigma_g \times S^1)$. We can therefore form $M = M_0 \cup W \cup (\Sigma_g \times D^2)$ and extend $f_0 : M_0 \to D^2$ via the obvious projections to a genus g Lefschetz fibration $f : M \to S^2$.

Assume now, moreover, that h_t is an isotopy through fiber-preserving homeomorphisms. By Lemma 2, there are homeomorphisms $k_t : S^2 \to S^2$ satisfying $k_t \circ \pi = \pi \circ h_t$. The map $\pi \times id \times id : \Sigma_g \times I \times I \to S^2 \times I \times I$ then descends to a well-defined quotient map

$$W = \frac{\Sigma_g \times I \times I}{(x,1,t) \sim (h_t(x),0,t)} \longrightarrow \frac{S^2 \times I \times I}{(\pi(x),1,t) \sim (k_t \circ \pi(x),0,t)} \cong S^2 \times S^1 \times I.$$

This map is a simple 3-fold branched cover, which by Remark 8 restricts to the branched covering of Theorem 7 on $\partial M_0 \subset \partial W$; we therefore have a cobordism of branched coverings from this branched covering to $\pi \times id$. We can use this cobordism to extend the branched covering of Theorem 7 to a simple 3-fold covering

$$M = M_0 \cup W \cup (\Sigma_g \times D^2) \longrightarrow (S^2 \times D^2) \cup (S^2 \times S^1 \times I) \cup (S^2 \times D^2),$$

whose base is an S^2-bundle over S^2. The branch set will be the closed surface obtained by attaching the $2g + 4$ disks $(P \cup R) \times D^2$ to the boundary of the ribbon manifold branch set described in Remark 9. Note that by Corollary 10, this branch set has at least three components. This is a contradiction: Siebert and Tian demonstrate ([10], Proposition 2.3) that the branch set for any nontrivial Lefschetz fibration obtained as a simple branched covering has at most two connected components. □

Remark 12. The result of Siebert and Tian that is used to produce the contradiction in the above proof relies crucially on the assumption that the Lefschetz fibrations under consideration are symplectic. See Remark 6. This raises the following question, about which our argument says nothing.

Question. Are global monodromies of achiral Lefschetz fibrations over S^2 fiber-isotopic to the identity?

Considering achiral Lefschetz fibrations has the effect of allowing the Dehn and disk twists appearing in the monodromies to be both left and righthanded. Thus, this question is equivalent to asking if a homeomorphism of the form $D^{\varepsilon_\mu}_{\gamma_\mu,\gamma_\mu'} \circ \cdots \circ D^{\varepsilon_1}_{\gamma_1,\gamma_1'}$ with each $\varepsilon_i \in \{1,-1\}$ can be fiber-isotopic to the identity.

Remark 13. The assumption in Theorem 7 and in the proof of Theorem 11 that all of the vanishing cycles are nonseparating was merely a convenience. By adapting the techniques in [3] to the simple 3-fold cover setting, Theorem 7 can be extended to allow Lefschetz fibrations with separating vanishing cycles. Since Siebert and Tian allow for both nonseparating and separating vanishing cycles in Proposition 2.3 of [10], our proof of Theorem 11 adapts easily to show that the global monodromies of Lefschetz fibrations containing separating vanishing cycles cannot be fiber-isotopic to the identity, as well.

Remark 14. There is a different construction–in a context removed from Lefschetz fibrations–of fiber-preserving homeomorphisms of Σ_g which are isotopic to the identity, but not fiber-isotopic to the identity. This construction is based on the observation that there are arcs δ in S^2 for which a disk twist about δ lifts to a composition of a Dehn twist about a simple closed curve which bounds a disk in Σ_g and a disk twist about an arc in Σ_g [5]. This lift is isotopic to the identity, but cannot be fiber-isotopic to the identity because it fails to fix the preimages under π of the branch set B [6]. We note that homeomorphisms of the form considered in Theorem 11 can be found which fix the preimages of the branch points.

Remark 15. The growing literature on the topology of Lefschetz fibrations provides explicit collections of curves giving depictions of the nonfiber-isotopic homeomorphisms constructed in Theorem 11. We refer the reader once again to [4] for a survey.

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School of Mathematics, Institute for Advanced Study, Olden Lane, Princeton, New Jersey 08540

Current address: Department of Mathematics, California State University, Northridge, California 91330

E-mail address: terry.fuller@csun.edu