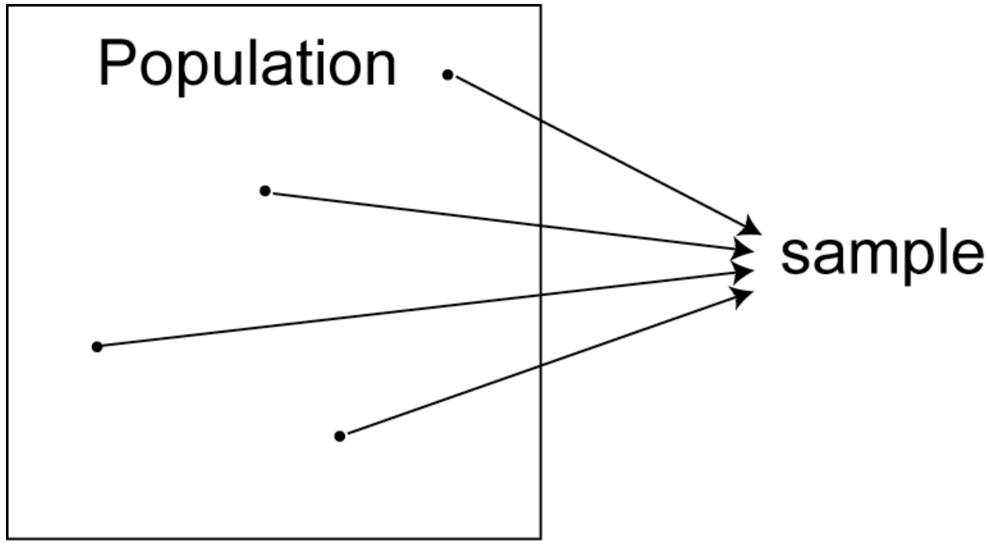
Simple random sample (SRS):



Why do we study Probability?

- Random samples eliminate bias (that’s good)
- Random samples will vary from sample to sample (that’s bad)

We study probability to tell us what will happen if we take very many samples.

Terminology:
The probability of an outcome is the theoretical proportion of times that outcome would occur in a very long sequence of repetitions.

Examples:

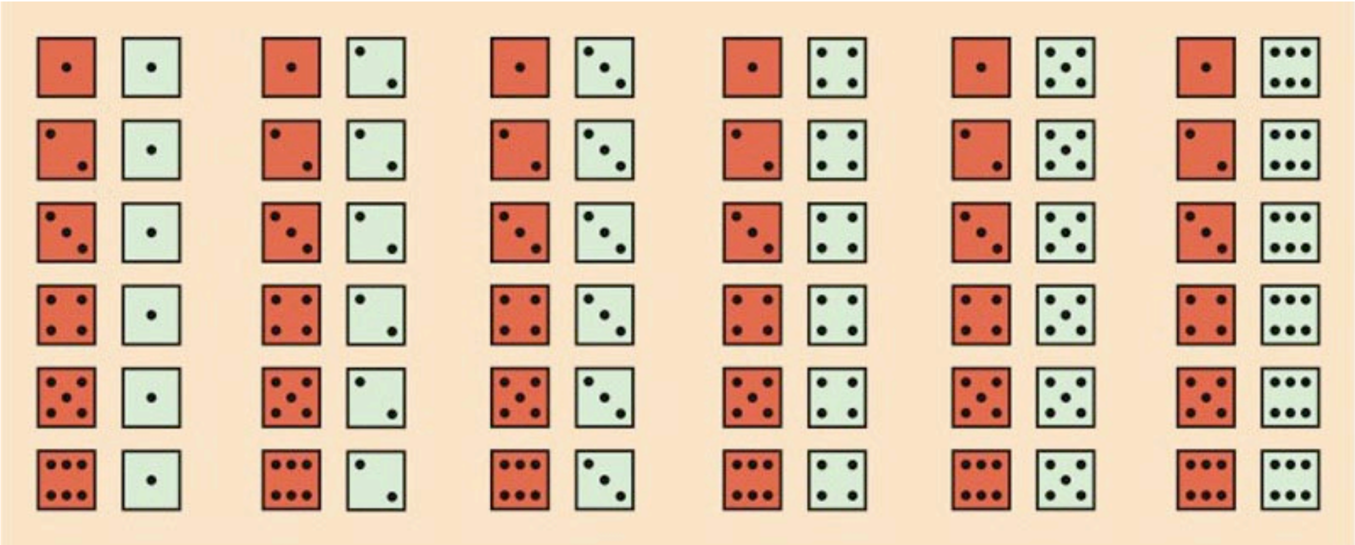
1. Toss coin: P(heads) = 1/2 = 0.5, P(tails) = 1/2 = 0.5

2. Roll dice: P("6") = 1/6 = 0.167

3. P(Shaq makes free throw) ≈ 0.52
   P(Kobe makes free throw) ≈ 0.84

Sample space S: set of all possible outcomes
Event: one or more possible outcomes

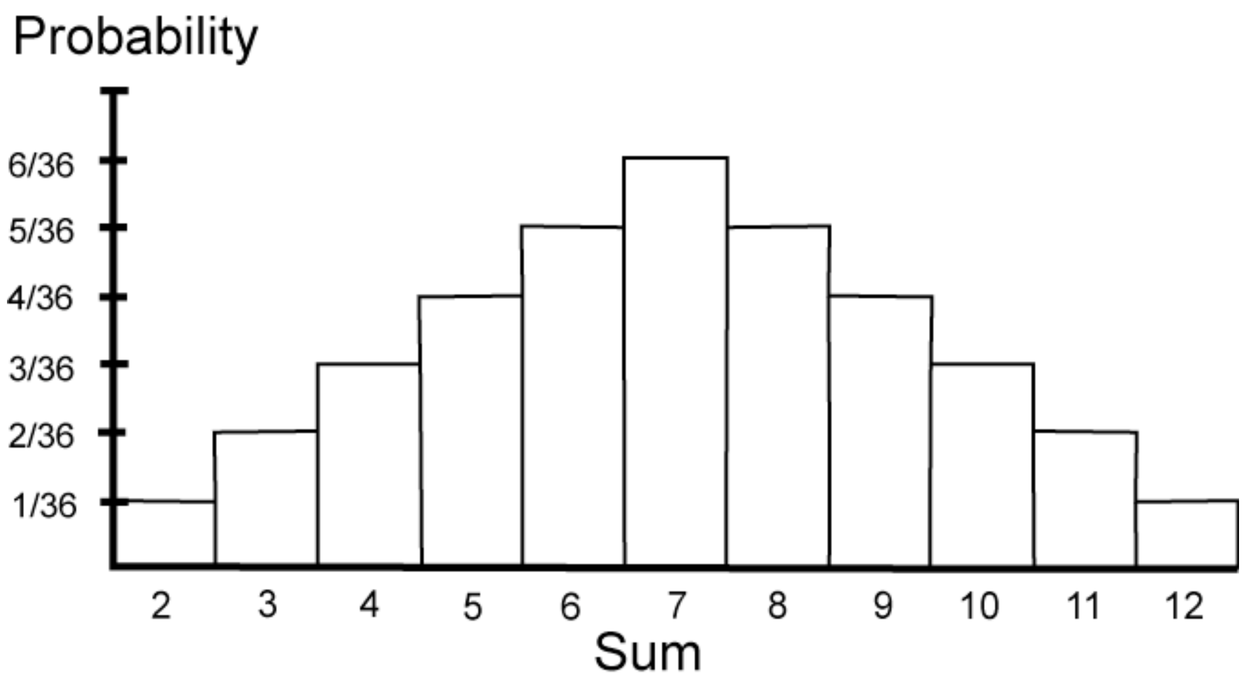
Example: roll two dice, add them together



S = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

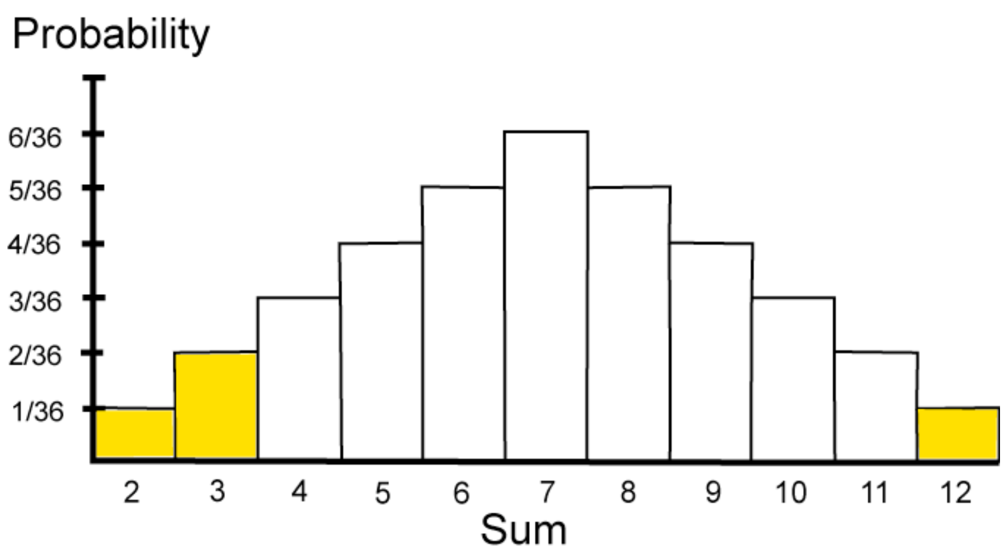
Sum	2	3	4	5	6	7	8	9	10	11	12
Probability	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

Sum	2	3	4	5	6	7	8	9	10	11	12
Probability	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36



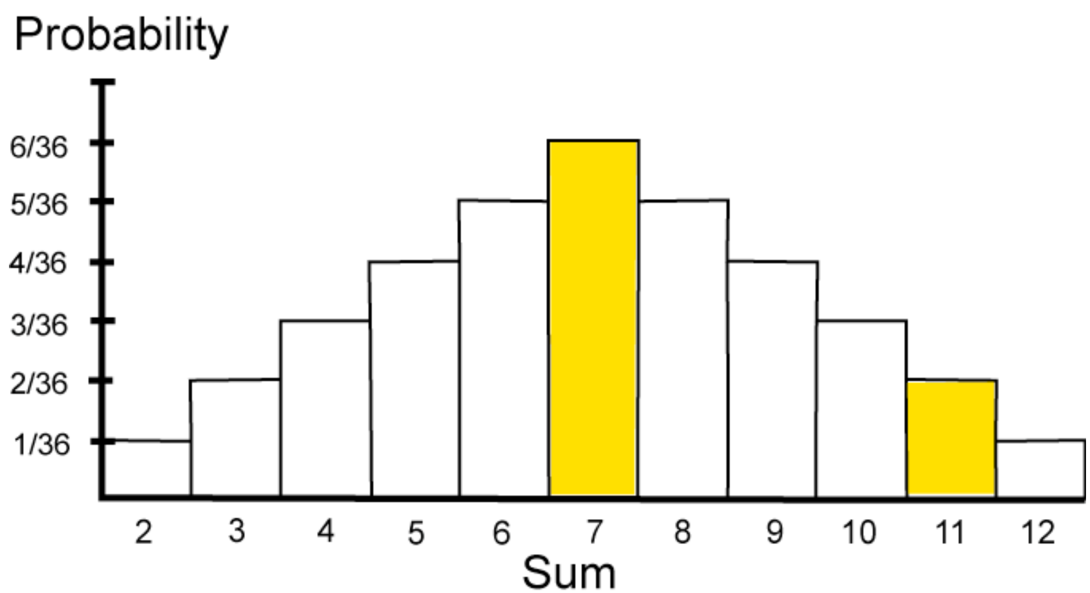
Notice: total area = 1.

- P("craps") = P(2, 3, or 12)
  = P(2) + P(3) + P(12)
  = 1/36 + 2/36 + 1/36 = 4/36 = 0.111



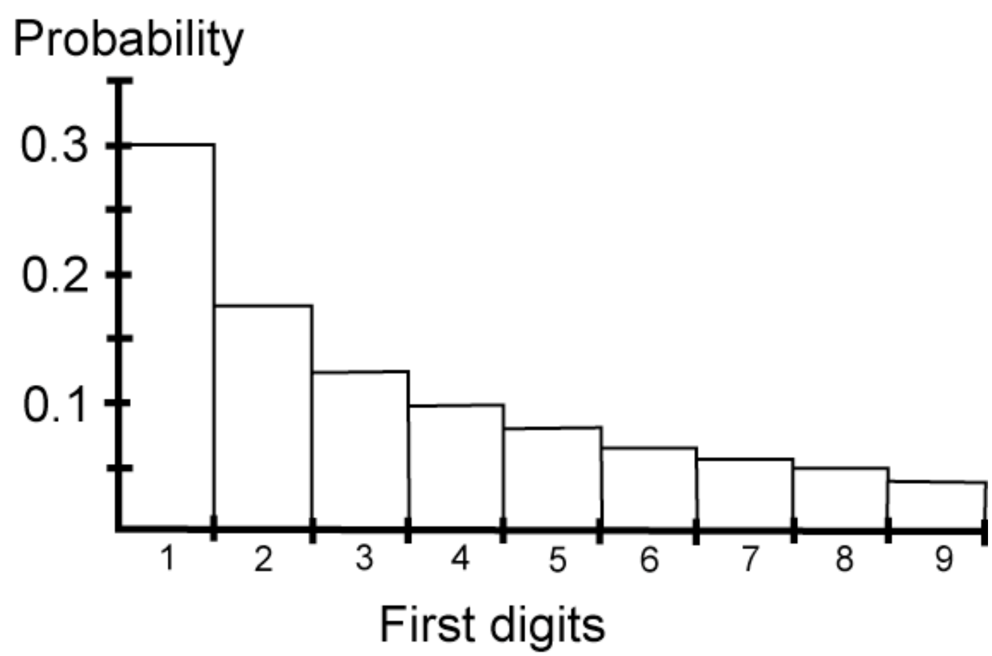
- P(not "craps")= 1 – 4/36 = 32/36 = 0.889

- $P(7 \text{ or } 11) = P(7) + P(11) = 6/36 + 2/36 = 8/36 = 0.222$



Example: Benford's law
X = first digit of numbers in legitimate financial records

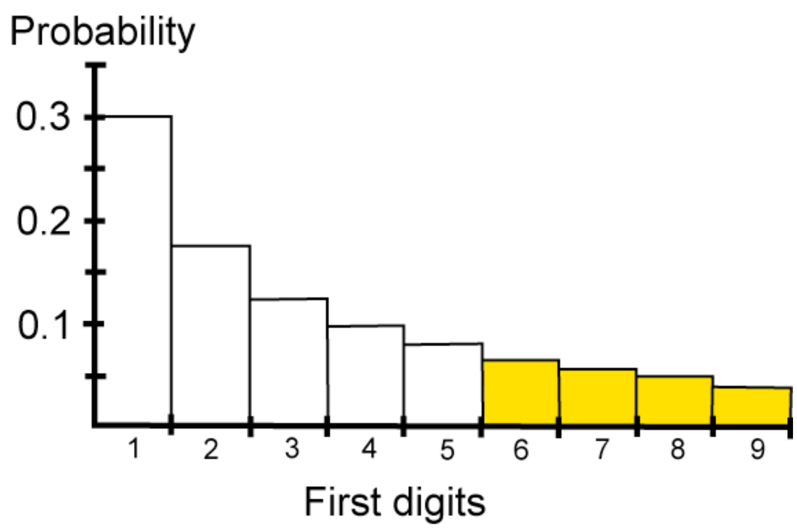
X	1	2	3	4	5	6	7	8	9
P(X)	0.301	0.176	0.125	0.097	0.079	0.067	0.058	0.051	0.046



X	1	2	3	4	5	6	7	8	9
P(X)	0.301	0.176	0.125	0.097	0.079	0.067	0.058	0.051	0.046

- Probability first digit greater than or equal to 6:

$$P(X \geq 6) = P(X=6) + P(X=7) + P(X=8) + P(X=9)$$
$$= 0.067 + 0.058 + 0.051 + 0.046$$
$$= 0.222$$



Probability Rules:

1. $0 \leq P(A) \leq 1$ for all events A.

2. For the entire sample space S, $P(S) = 1$.

3. If A and B are disjoint events (no common outcomes), then $P(A \text{ or } B) = P(A) + P(B)$.

4. $P(A \text{ does not occur}) = 1 – P(A)$.

We refer to mathematical descriptions of random phenomena as probability models or probability distributions.

Two kinds of probability models. So far, our examples are …

1st kind: Sample space finite → **discrete** probability model:
list the probabilities of all individual outcomes.

For a discrete model to be valid:
- individual probabilities must all be between 0 and 1
- the sum of all individual probabilities is 1.

Probabilities can be interpreted as areas in a bar graph.

2nd kind of model:

Sample space infinite → **Continuous** probability model

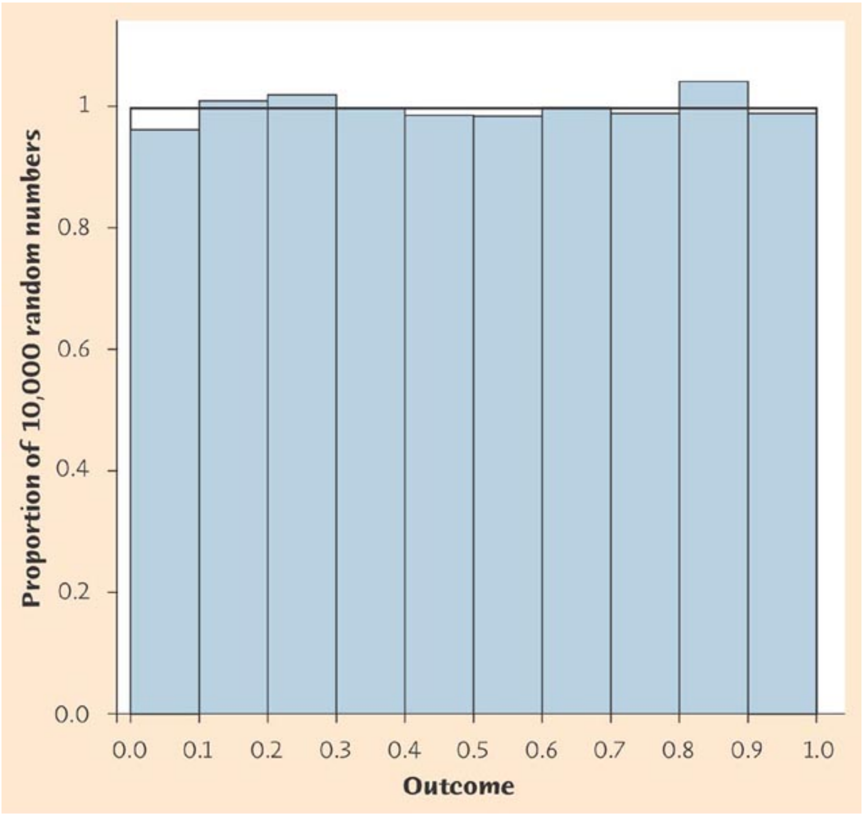
Examples:

- heights
- random number generator, *any* number between 0 and 1

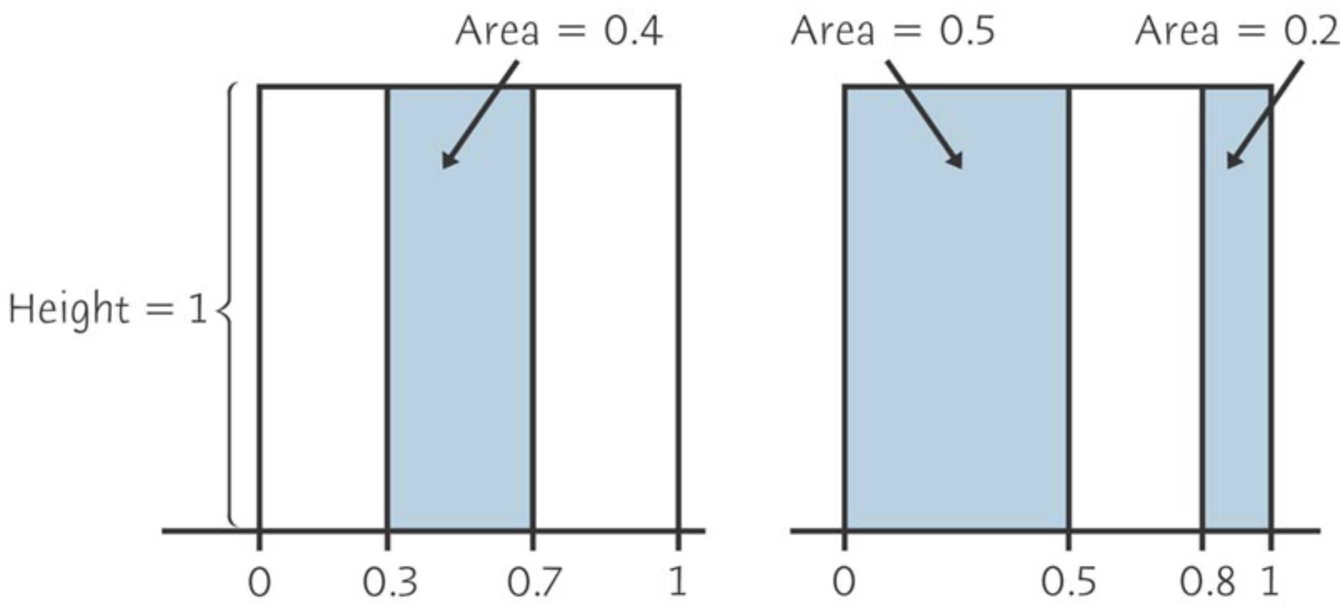
For continuous probability models, we find probabilities as Areas under a density curve.

Example: Choose *any* number Y between 0 and 1 at random.

Computer simulation picked 10,000 random numbers:



Example: Choose *any* number Y between 0 and 1 at random.



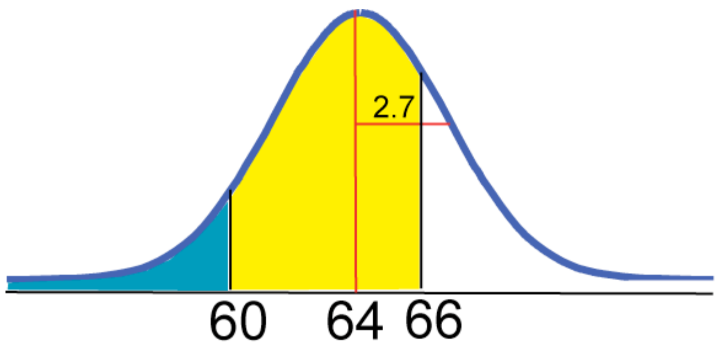
(a) $P(0.3 \leq Y \leq 0.7)$ (b) $P(Y \leq 0.5$ or $Y > 0.8)$

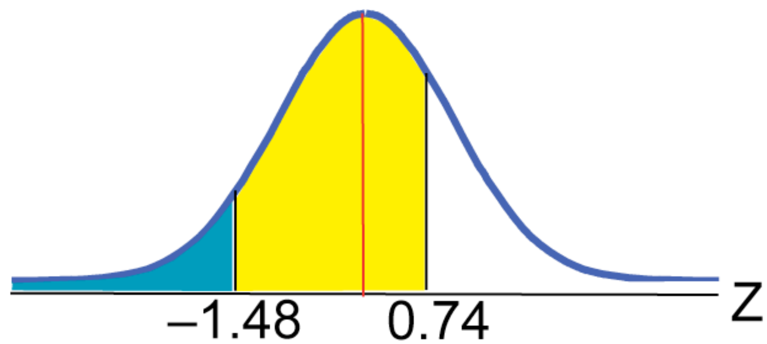
So (a) $P(0.3 \leq Y \leq 0.7) = 0.4$

(b) $P(Y \leq 0.5$ or $Y > 0.8) = 0.5 + 0.2 = 0.7$

Normal distributions give probability models.

Example: Heights of young women ~ N(64, 2.7) inches
What is the probability a randomly chosen women is
(a) shorter than 60 inches tall?
(b) between 60 and 66 inches tall?





- $z = (66–64)/2.7 = 0.74 \rightarrow$ Table A area: 0.7704
- $z = (60–64)/2.7 = –1.48 \rightarrow$ Table A area: 0.0694

(a) $P(X < 60) = 0.0694$
(b) $P(60 \leq X \leq 66) = 0.7704 – 0.0694 = 0.7010$