

Example: Does pre-school help people avoid needing social services (e.g. Welfare) later in life?

Sample:

- 61 children no pre-school, 49 needed social services

$$\hat{p}_1 = \frac{49}{61} = 0.803$$

- 62 children pre-school, 38 needed social services

$$\hat{p}_2 = \frac{38}{62} = 0.613$$

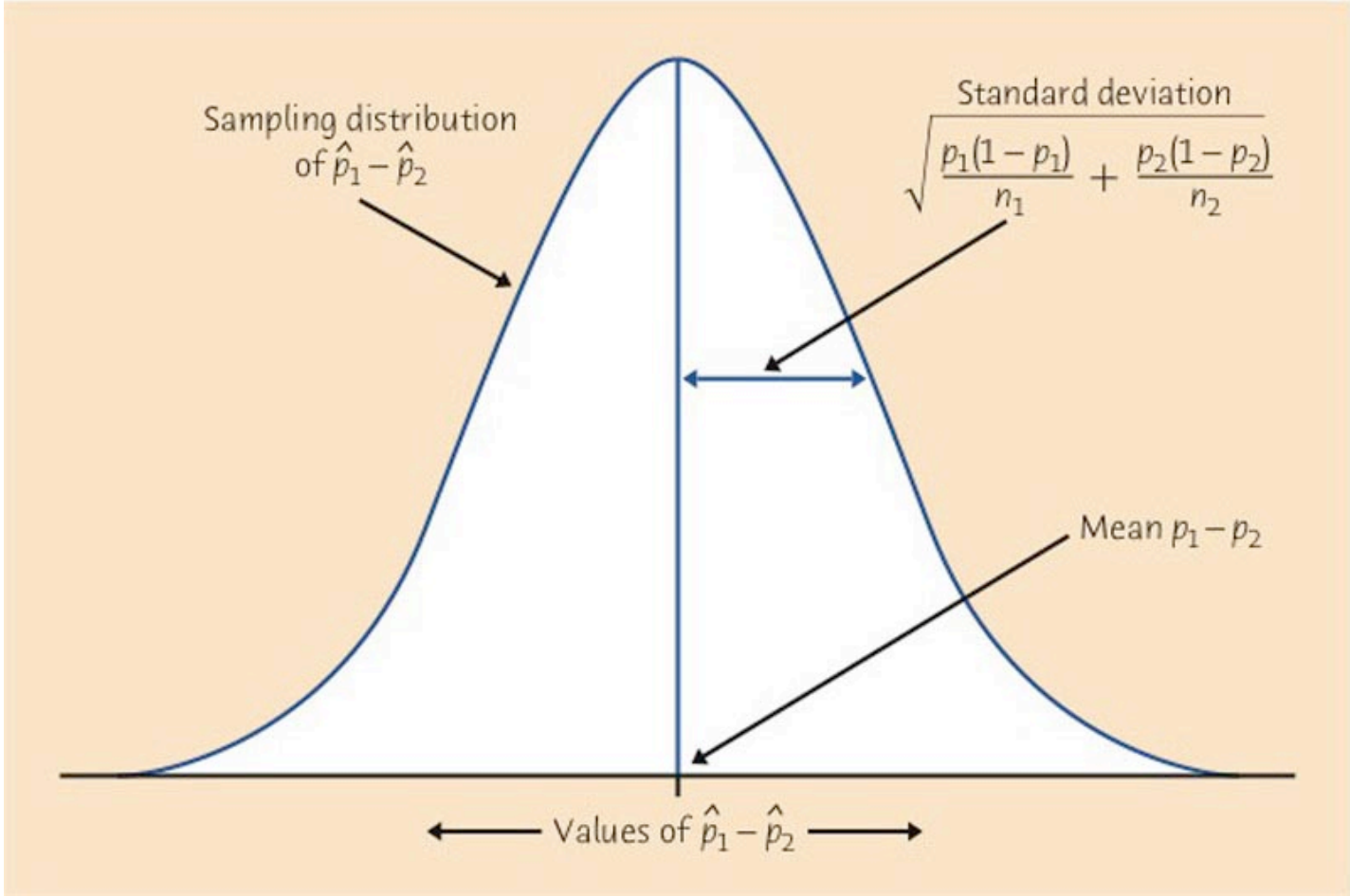
Example: Would you date someone with a great personality even though you don't find them attractive? You said ...

• Men: Yes 9
 No 8
 (Total 17)

$$\hat{p}_1 = \frac{9}{17} = 0.53$$

• Women Yes 9
 No 4
 (Total 13)

$$\hat{p}_2 = \frac{9}{13} = 0.69$$



Level C confidence interval for $p_1 - p_2$ is

$$(\hat{p}_1 - \hat{p}_2) \pm z^* SE$$

where the standard error SE is

$$SE = \sqrt{\frac{\hat{p}_1(1 - \hat{p}_1)}{n_1} + \frac{\hat{p}_2(1 - \hat{p}_2)}{n_2}}$$

Example: Give a 95% confidence interval for the difference in the proportion of non-preschool versus preschool students needing social services.

Sample:

- 61 children no pre-school, 49 needed social services

$$\hat{p}_1 = \frac{49}{61} = 0.803$$

- 62 children pre-school, 38 needed social services

$$\hat{p}_2 = \frac{38}{62} = 0.613$$

$$SE = \sqrt{\frac{(.803)(.197)}{61} + \frac{(.613)(.387)}{62}} = 0.080122$$

$$\begin{aligned} (0.803 - 0.613) \pm (1.96)(0.080122) &= 0.190 \pm 0.157 \\ &= (0.033, 0.347) \end{aligned}$$

We are 95% confident that the proportion of non-preschool students needing social services is between 3.3 and 34.7% higher than that of preschool students.

Example: Would you date someone with a great personality even though you don't find them attractive? Give 90% confidence interval for the difference between Men and Women.

- Men: $\hat{p}_1 = \frac{46}{92} = 0.50$

- Women: $\hat{p}_2 = \frac{70}{90} = 0.78$

$$SE = \sqrt{\frac{(.50)(.50)}{92} + \frac{(.78)(.22)}{90}} = 0.068$$

$$(0.50 - 0.78) \pm (1.645)(0.068) = -0.28 \pm 0.112$$
$$= (-0.392, -0.168)$$

We are 90% confident that the proportion of men willing to date somebody they don't find attractive is between 39.2% and 16.8% lower than the proportion for women.

Significance tests for comparing two Proportions:

To test $H_0: p_1 = p_2$,

use the test statistic

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1 - \hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where \hat{p} is the "pooled" proportion of both samples combined.

Example: Are these samples evidence that men are less likely to date somebody they don't find attractive?

• Men: $\hat{p}_1 = \frac{46}{92} = 0.50$

• Pooled: $\hat{p} = \frac{46 + 70}{92 + 90} = \frac{116}{182} = 0.64$

• Women: $\hat{p}_2 = \frac{70}{90} = 0.78$

$$H_0: p_1 = p_2$$

$$H_a: p_1 < p_2$$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.50 - 0.78}{\sqrt{(0.64)(0.36)\left(\frac{1}{90} + \frac{1}{92}\right)}} = -3.78$$

$$P < 0.0002$$

This is strong evidence that men are less likely than women to date somebody they don't find attractive.

Calculator tip: use the x^{-1} key for reciprocals

$$\frac{0.50 - 0.78}{\sqrt{(0.64)(0.36)\left(\frac{1}{90} + \frac{1}{92}\right)}}$$

Type: “(.50-.78) ÷ √(.64 × .36 × (90⁻¹+92⁻¹))”

Example: Do older people snore more than younger people?

• Young: $\hat{p}_1 = \frac{48}{184} = 0.261$

• Pooled: $\hat{p} = \frac{48 + 318}{184 + 811} = 0.368$

• Old: $\hat{p}_2 = \frac{318}{811} = 0.392$

$$H_0: p_1 = p_2$$

$$H_a: p_1 < p_2$$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1 - \hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.261 - 0.392}{\sqrt{(0.368)(0.632)\left(\frac{1}{811} + \frac{1}{184}\right)}} = -3.33$$

$P = 0.0004$. This is very strong evidence that older people snore more.

Using these procedures for 2-sample Proportions:

- *Confidence intervals*: use if the number of successes and the number of failures in each sample are at least 10.
- *Significance Tests*: use if number of successes and the number of failures in each sample are at least 5.

Using the TI 83/84:

- confidence intervals **STAT** → **TESTS** → **B:2-PropZInt**
- significance tests **STAT** → **TESTS** → **6:2-PropZTest**