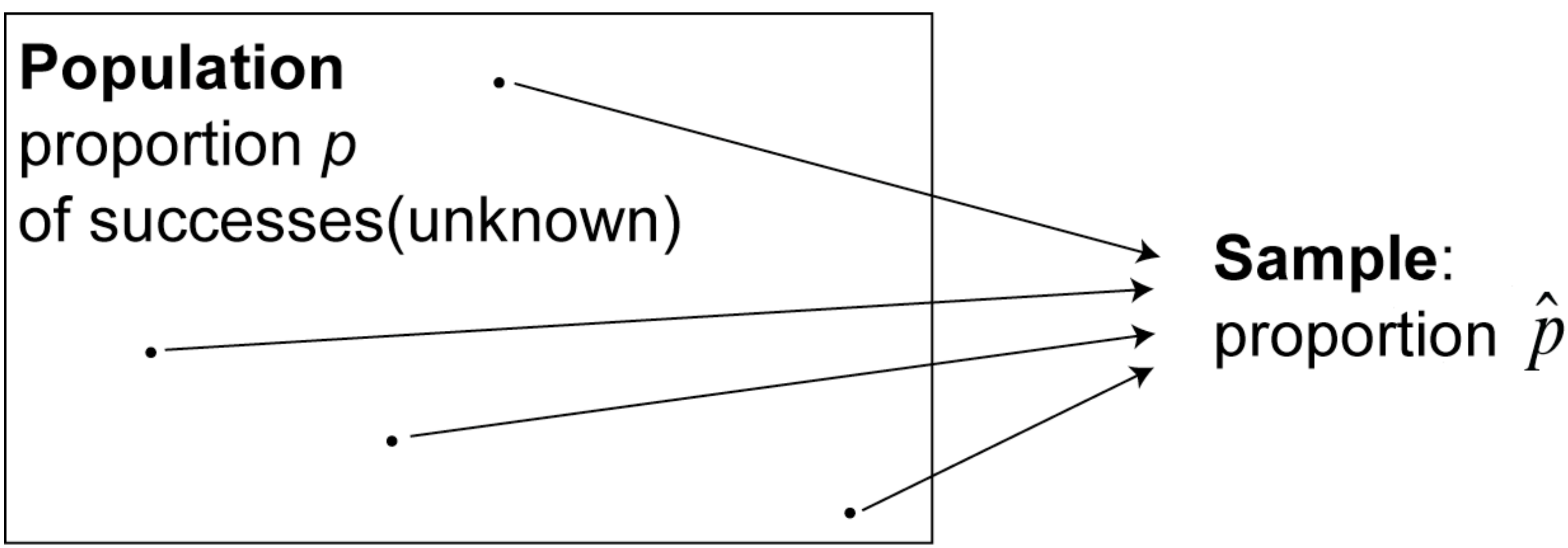
Inference for Proportions:

- (book) sample of 2673 adult heterosexuals, 170 had more than one sexual partner (in the last year)

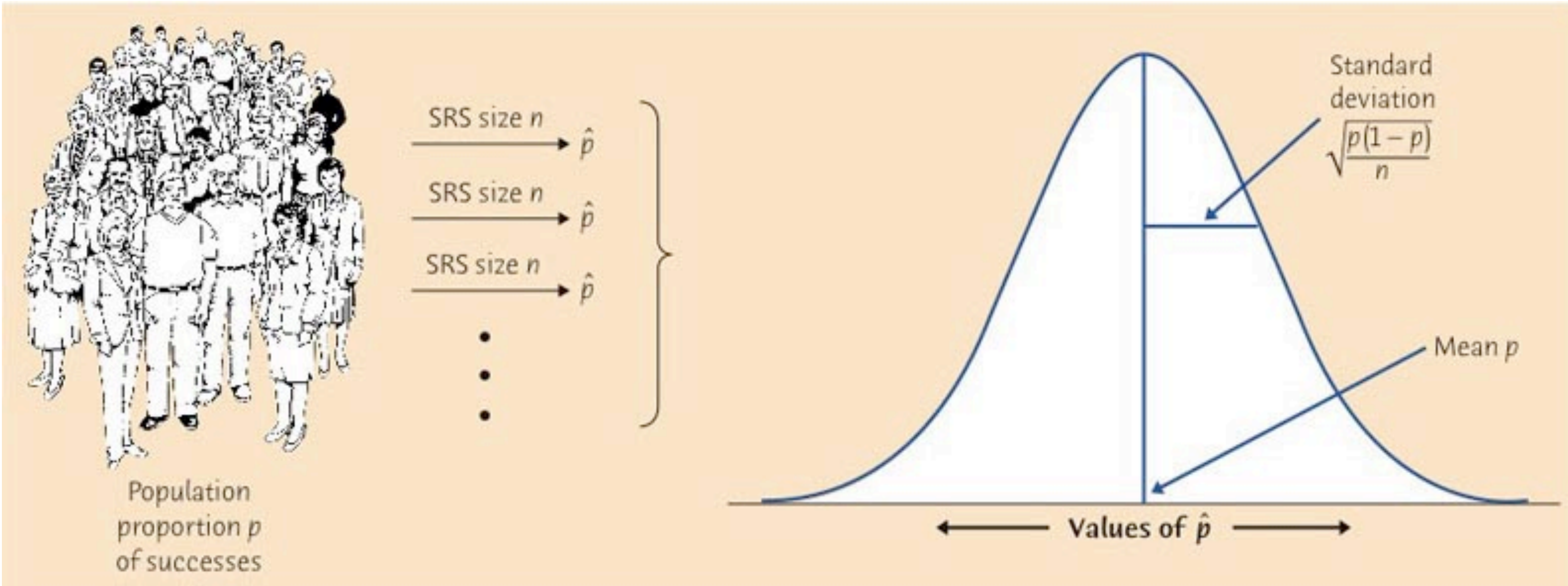
$$\hat{p} = \frac{170}{2673} = 0.0636$$

- Math 140 1st day of class, 15 out of 212 left-handed

$$\hat{p} = \frac{15}{212} = 0.071$$



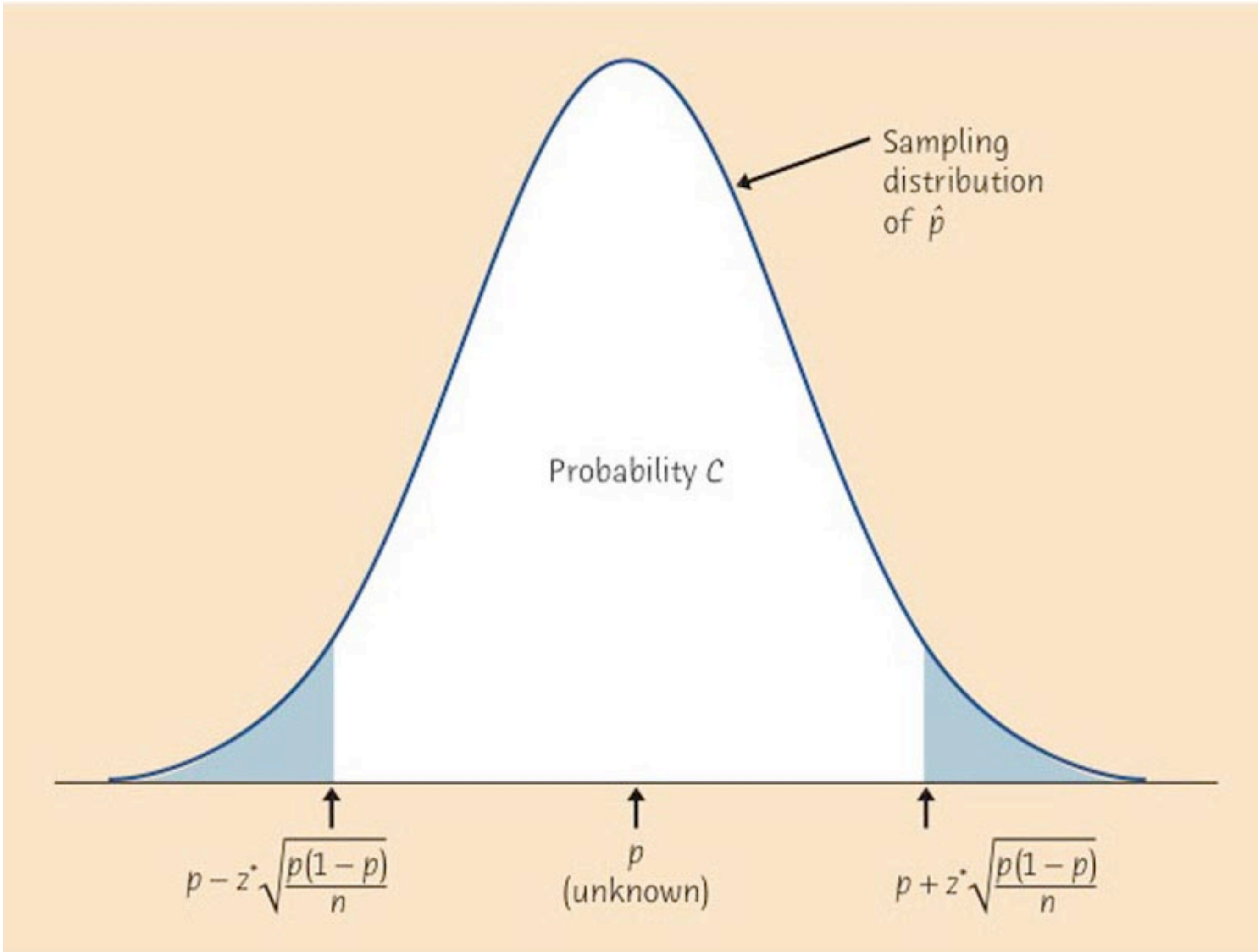
Sampling Distribution for Proportions:



The distribution of $\hat{p}$ is approximately normal with

- mean p
- standard deviation $\sqrt{\frac{p(1-p)}{n}}$

C% Confidence Interval:



So the level C confidence interval is given by

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

where z^* is the critical value with area C between $-z^*$ and z^* on the standard normal distribution.

- We use $\hat{p}$ to approximate p
- We use standard normal distribution z, not t. We use t distributions only for means.

Example: sample of 2673 adult heterosexuals, 170 had more than one sexual partner (in the last year). Give 99% confidence interval for the true proportion for all adult heterosexuals.

$$\hat{p} = \frac{170}{2673} = 0.0636$$

99% confidence → $z^* = 2.576$

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.0636 \pm (2.576)\sqrt{\frac{(0.0636)(0.9364)}{2673}}$$

$$= 0.0636 \pm 0.0122 = (0.0514, 0.0758)$$

We are 99% confident the true proportion of all adult heterosexuals with more than one sexual partner is between 0.0514 and 0.0758.

Example: Math 140 1st day of class, 8 out of 110 left-handed. Give 95% percent confidence interval for the true proportion of left-handed people in statistics classes.

$$\hat{p} = \frac{8}{110} = 0.07 \quad \text{(rounded)}$$

95% → z* = 1.960

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.07 \pm (1.96)\sqrt{\frac{(0.07)(0.93)}{110}}$$

$$= 0.07 \pm 0.048 = (0.022, 0.118)$$

We are 95% confident the true proportion of left-handed people taking statistics is between 1.2% and 10.8%

Example: October 30, 2008 *CNN/NY Times* poll of 1439 likely voters, 51% said they would vote for Barack Obama.
Give 95% confidence interval for the true proportion of Obama voters nationwide.

$$\hat{p} = 0.51$$

$$0.51 \pm (1.96)\sqrt{\frac{(0.51)(0.49)}{1439}} = 0.51 \pm 0.03 = (0.48,\ 0.54)$$

We were 95% confident that true proportion of Obama voters is between 48% and 54% percent.

(In the actual election, p = 53%)

Example (continued): What sample size would be needed to reduce the margin of error in the previous example to 0.01?

$$(1.96)\sqrt{\frac{(0.51)(0.49)}{n}} = 0.01$$

$$\frac{1.96}{0.01}\sqrt{(0.51)(0.49)} = \sqrt{n}$$

$$n = \left(\frac{1.96}{0.01}\right)^2 (0.51)(0.49) = 9600.2$$

So you need a sample size of 9601 or more to have a margin of error below 0.01.

- Confidence intervals for proportions should only be used when the number of “successes” and “failures” in the sample are both at least 15.

- Confidence intervals on the TI 83/84:

  STAT → TESTS → A:1-PropZInt

  1-PropZInt
  x: 170
  n: 2673
  C-Level: .99
  Calculate (enter)

- Finding the sample size *n* for desired margin of error *m*:

Solve for n:

$$z^* \sqrt{\frac{(p^*)(1-p^*)}{n}} = m$$

For *p**, use either

- an educated guess from previous sample
- conservative estimate of *p** = 0.5

Example: Are babies more likely to be boys than girls? Sample of 25,468 babies had 13,173 boys.

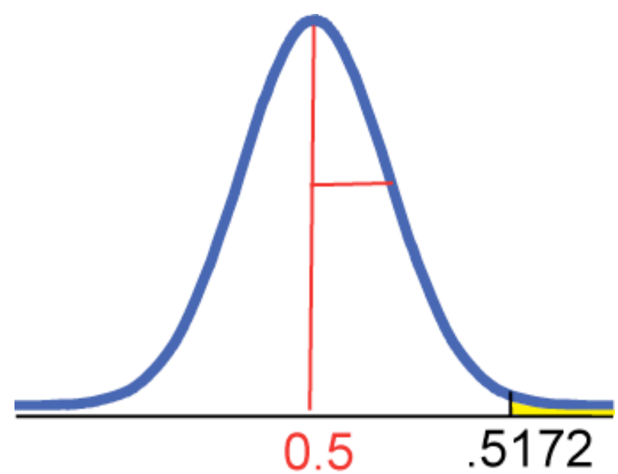
$$\hat{p} = \frac{13{,}173}{25{,}468} = 0.5172$$

Is this evidence that babies are more likely to be boys?

Assume H_0: $p = 0.5$

H_a: $p > 0.5$

$$z = \frac{0.5172 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{25{,}468}}} = 5.49$$



P < 0.0002, so this is very strong evidence that more than half of babies are boys.

Example: Sample of Math 140 classes, 15 out of 212 left-handed.

$$\hat{p} = \frac{15}{212} = 0.071$$

Is this evidence that statistics students have a different proportion of left-handedness than the general population (which is 10%)?

H_0: p = 0.10
H_a: p ≠ 0.10

$$z = \frac{0.071 - 0.10}{\sqrt{\frac{(0.1)(0.9)}{212}}} = -1.41$$

P = (2)(0.0793) = 0.1586, so this is not evidence that statistics students have a different proportion of left-handedness than the general population.

Example: Is home court advantage real?

NBA regular season: home team won 727 out of 1230 games.

$$\hat{p} = \frac{727}{1230} = 0.59$$

Is this evidence for home court advantage?

H_0: p = 0.50
H_a: p > 0.50

$$z = \frac{0.59 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{1230}}} = 6.31$$

P < 0.0002, so this is strong evidence for home court advantage.

- Significance tests for proportions should only be used when the number of “successes” and “failures” in the sample are both at least 10.

- Significance tests on the TI 83/84:

STAT → TESTS → 5:1-PropZTest

1-PropZTest

p_0: .5

x: 727

n: 1230

prop $\neq p_0$ $< p_0$ **$> p_0$**

Calculate   Draw (enter)