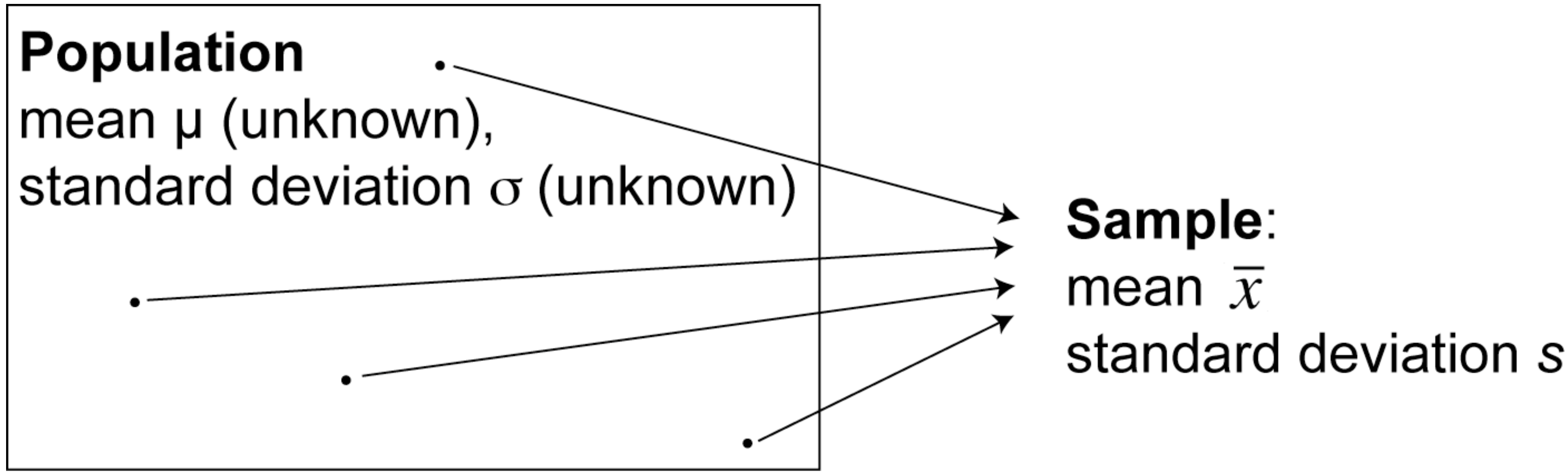


Statistical Inference:

- Confidence Intervals
- Significance Tests

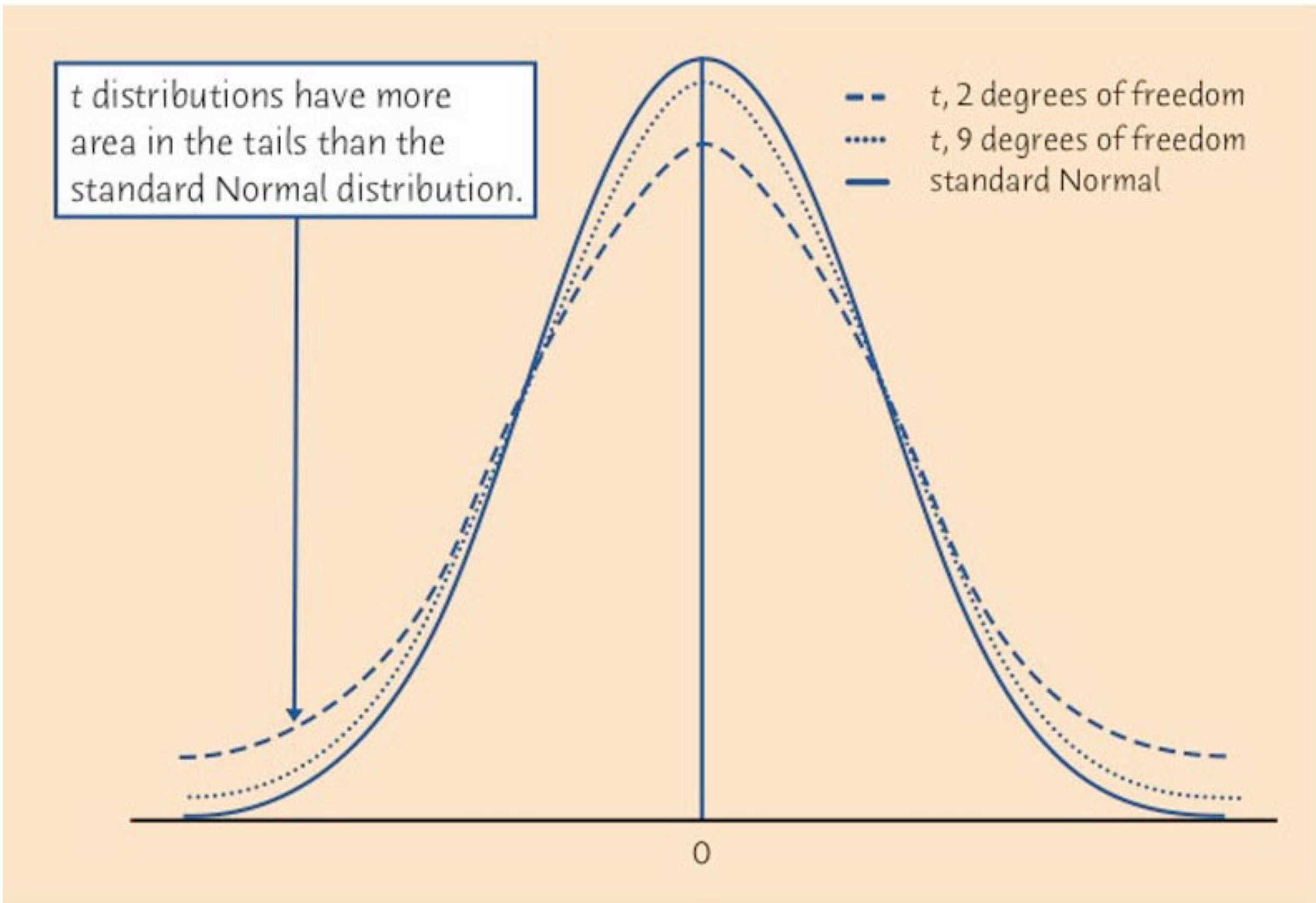
What do we do when we don't know σ ?

(Partial) answer: Use *s* from sample in place of σ.

But there is a problem ...

... when we use s to estimate σ, we do not get a Normal distribution.

We get a t distribution with n–1 degrees of freedom (t(n-1))



A level C confidence interval for μ is $\bar{x} \pm t^* \frac{s}{\sqrt{n}}$.

The critical value t* is chosen so that the curve *t(n-1)* has area C between –t* and t*.

Terminology: $\frac{s}{\sqrt{n}}$ is called the Standard Error (SE)

Formula sometimes written: $\bar{x} \pm t^* SE$.

Example: rate of skin healing in newts.
(units: micrometers per hour)

29	27	34	40	22	28	14	35	26
35	12	30	23	18	11	22	23	33

What is a 95% confidence interval for the rate of skin healing for all newts?

$$\bar{x} = \frac{29 + 27 + \cdots + 33}{18} = 25.67 \qquad s = Sx = 8.324$$

Table C: confidence 95% (column)
degrees of freedom df = 17 (row) → t*=2.110

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}} = 25.67 \pm 2.110 \frac{8.324}{\sqrt{18}} = 25.67 \pm 4.14 = (21.53, 29.81)$$

We are 95% confident that the mean rate of skin healing for all newts is between 21.53 and 29.81 micrometers/hour.

Example: Sample of waiting times for ride at Disney World:

30	29	34	41	37	16	18
30	24	25	29	16	19	30

Give 90% confidence interval for the mean waiting time (for everybody).

$$\text{Sample}: \quad \bar{x} = 27.00 \qquad s = 7.75$$

df = 14 – 1 = 13 $\rightarrow$ t* = 1.771

$$\text{Standard error}: \quad SE = 7.75/\sqrt{14} = 2.071$$

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}} = 27.00 \pm 1.771 \frac{7.75}{\sqrt{14}} = 27.00 \pm 3.67 = (23.33, 30.67)$$

We are 90% confident the mean waiting time is between 23.33 and 30.67 minutes.

Example: Do colas lose sweetness during storage?

Experiment: trained testers compared colas before and after storage, ranking sweetness on scale of 1 to 10.

before – after:	2.0	0.4	0.7	2.0	–0.4	2.2	–1.3	1.2	1.1	2.3

$$\bar{x} = \frac{2.0 + 0.4 + \cdots + 2.3}{10} = 1.02 \qquad s = 1.196$$

Is this evidence that the colas lose sweetness during storage?

H_0: $\mu = 0$
H_a: $\mu > 0$

$$t = \frac{1.02 - 0}{1.196/\sqrt{10}} = 2.697$$

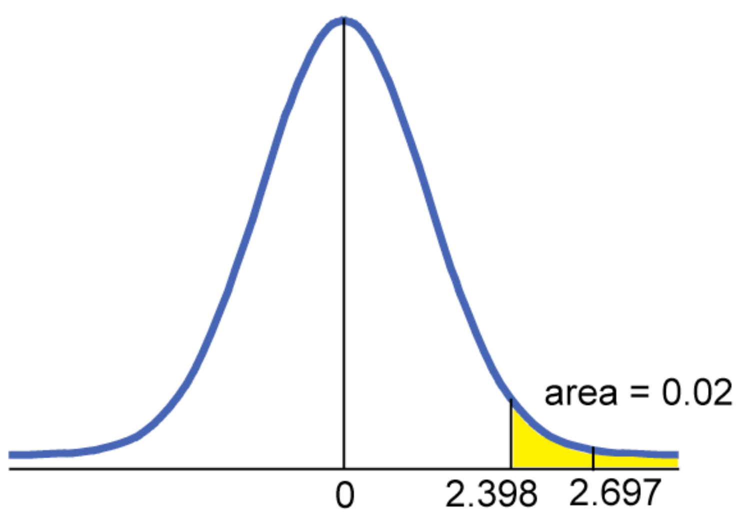
To find P-value for t(9): Table C gives upper tail areas for some useful values.

we calculated t = 2.697

df=9 t*	2.398	2.821
One-sided P	.02	.01

This says P-value between 0.01 and 0.02. In particular, P < 0.02. So this is evidence that colas lose sweetness.

t-distribution, 9 degrees of freedom



Example: A bottling machine at brewery is suspected of under-filling beer bottles. (Label says 12 ounces.)

Sample of 16 bottles measured, $\bar{x} = 11.83$ ounces, $s = 0.178$.

Is this evidence the machine is under-filling?

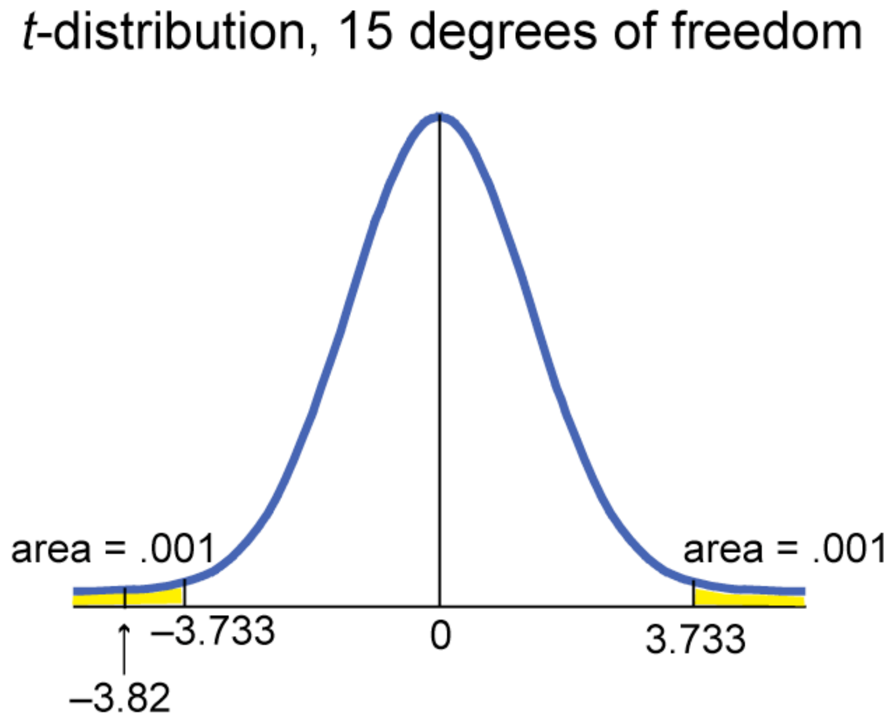
Assume H_0: μ = 12
H_a: μ < 12

$$t = \frac{11.83 - 12}{0.178 / \sqrt{16}} = -3.82$$

df=15 t*	3.733	4.073
One-sided P	.001	.0005

P-value is between .0005 and .001.
This is strong evidence the machine is under-filling.

df=15 t*	3.733	4.073
One-sided P	.001	.0005



So P-value is less than .001.

Example: Is 98.6˚F really the mean human body temp? Or is it different from 98.6˚F?

H_0: μ = 98.6
H_a: μ ≠ 98.6

Sample of 52 people, $\bar{x} = 98.285$, $s = 0.6824$.

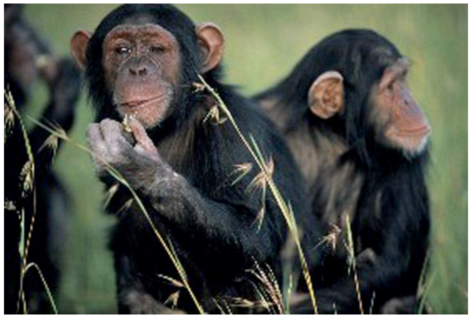
$$t = \frac{98.285 - 98.6}{0.6824/\sqrt{52}} = -3.33$$

df = 51; use df = 50 in Table C.

df=50 t*	3.261	3.496
Two-sided P	.002	.001

So P-value is between .001 and .002. This is evidence that body temperature is different from 98.6˚F.

Example: Are chimpanzees more likely to cooperate when it's necessary?



Matched pairs experiment: 8 chimps could recruit a partner to help them get food by pulling on two ropes.
For some trials cooperation was necessary, for others not.

TABLE 17.1 Trials (out of 24) on which chimpanzees recruited a partner

	COLLABORATION NEEDED		
CHIMPANZEE	YES	NO	DIFFERENCE
Namuiska	16	0	16
Kalema	16	1	15
Okech	23	5	18
Baluku	19	3	16
Umugenzi	15	4	11
Indi	20	9	11
Bili	24	16	8
Asega	24	20	4

TABLE 17.1 Trials (out of 24) on which chimpanzees recruited a partner

	COLLABORATION NEEDED		
CHIMPANZEE	YES	NO	DIFFERENCE
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Indi	20	9	11
Bili	24	16	8
Asega	24	20	4

Sample differences: $\bar{x} = 12.375,\ s = 4.749.$

Is this evidence that chimps are more likely to cooperate when it is necessary?

H_0: μ = 0
H_a: μ > 0

$$t = \frac{12.375 - 0}{4.749/\sqrt{8}} = 7.37$$

H_0: $\mu = 0$
H_a: $\mu > 0$

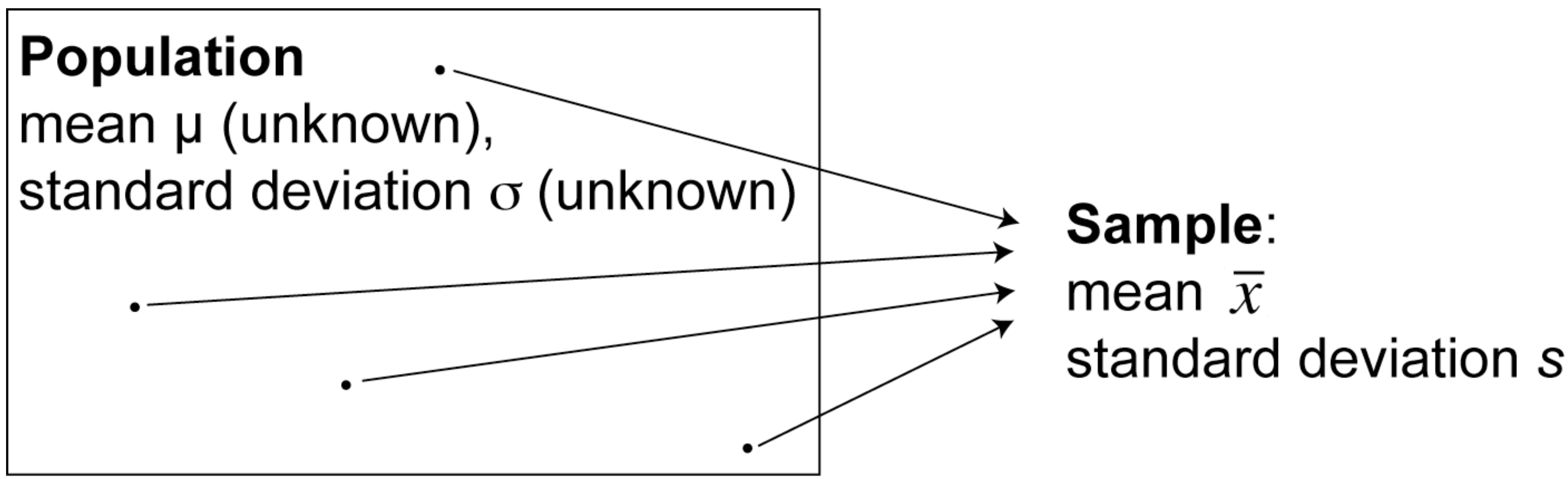
$$t = \frac{12.375 - 0}{4.749/\sqrt{8}} = 7.37$$

df=7 t*	4.785	5.408
One-sided P	.001	.0005

This says $P < 0.0005$

This is very strong evidence that chimpanzees are more likely to cooperate when it is necessary.

Summarizing:



What do we do when we don't know σ ?

1. Use s from sample in place of σ.
2. Work with (normalized) t-distribution with df = n-1 instead of standard normal distribution.

Using the *t* Procedures:

- *Sample size less than 15*: Use if data appear close to Normal. If data is skewed, or outliers, do not use.

- *Sample size at least 15*: The t procedures can be used except if outliers or strong skewness.

- *Sample size large (roughly n≥40)*: The t procedures can used even for skewed distributions.

Confidence intervals on the TI-83/84:
(Disney example, two ways)

STAT → TESTS → 8:TInterval (enter)

TInterval
Inpt: **Data** or Stats (enter)
List: L_1
Freq: 1
C-Level: .90
Calculate (enter)

TInterval
Inpt: Data or **Stats** (enter)
$\bar{x}$: 27.00
Sx: 7.75
n: 14
C-Level: .90
Calculate (enter)

Significance tests on the TI-83/84:
(Chimpanzee example, two ways)

STAT → TESTS → 2:T-Test (enter)

T-Test
Inpt: **Data** or Stats (enter)
μ_0: 0
List: L_1
Freq: 1
μ:$\neq\mu_0$ $<\mu_0$ **$>\mu_0$** (enter)
Calculate   **Draw** (enter)

T-test
Inpt: Data or **Stats** (enter)
μ_0: 0
$\bar{x}$: 12.375
Sx: 4.749
n: 8
μ:$\neq\mu_0$ $<\mu_0$ **$>\mu_0$** (enter)
Calculate   **Draw** (enter)