The code

Write a program, using your (preferred language) to implement Gauss factorization with partial pivoting. The data of the code are: the matrix A and the vector b.
The code must include a procedure for:

- reading A and b.
- finding the pivot. Such procedure must consider the case where A is singular.
- permuting the rows.
- performing the elimination.
- solving the upper triangular system.
- printing out the result i.e. the solution of the system.

Caution: Do not use n matrices since once A^{k+1} is computed, you do not need anymore A^k.

The validation

In order to validate your code, use the following strategy;

1. apply it to solve the following upper triangular system to validate the solving procedure.

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 3 & 2 \\ 0 & 0 & 4 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 7 \\ 12 \\ 12 \end{bmatrix}$$

2. apply it to the following system to validate the elimination procedure.

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 5 & -3 \\ 3 & 1 & 8 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 12 \\ 13 \\ 12 \end{bmatrix}$$

3. apply it to the following system to validate the procedure for finding the pivot as well as the permutation procedure.

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 3 & 1 \\ 4 & 1 & 1 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

4. Apply it to the following system and comment the result

$$A = \begin{bmatrix} 7 & 2 & -3 \\ 3 & 1 & 8 \\ 10 & 3 & 5 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Bonus question. Write a Gauss elimination solver for sparse matrices.