ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. *A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit.* Textbooks, classnotes, crib sheets, or calculators are not permitted.

1. (40 points) Consider the matrix

$$M = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ 0 & 5 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

   Find the QR factorization of M.

2. (30 points) For two invertible $n \times n$ matrices A and B, determine which of the following formulas are necessarily **TRUE**. You do not need to show your work.

   a. $(I_n - A)(I_n + A) = I_n - A^2$

   b. $(A + B)^2 = A^2 + 2AB + B^2$

   c. $(A^2)^{-1} = (A^{-1})^2$

   d. $(A + B)^{-1} = A^{-1} + B^{-1}$

   e. $rank(A) = rank A^{-1} = n$

3. (30 points) Consider the matrix

$$M = \begin{bmatrix} 0 & 2 & 1 & 0 & 1 \\ 0 & 0 & 2 & 0 & 2 \\ 0 & 5 & 3 & 9 & 9 \\ 0 & 7 & 4 & 0 & 1 \\ 3 & 9 & 5 & 4 & 8 \end{bmatrix}$$

   Find the determinant of M using Gauss-Jordan elimination.

HEY, THERE'S MORE—TURN THE PAGE OVER!

4. (40 points) Consider the transformation $T(A) = S^{-1}AS$ from $\mathbf{R}^{2\times 2}$ to $\mathbf{R}^{2\times 2}$, where

$$S = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$$

a. Show that the transformation T is linear.

b. Find the kernel and the nullity of the transformation T.

c. Use part (b) to find the rank of the transformation T.

d. Is the transformation T an isomorphism?

5. (30 points) Consider the matrix

$$A = \begin{bmatrix} -3 & 0 & 4 \\ 0 & -1 & 0 \\ -2 & 7 & 3 \end{bmatrix}$$

a. Find all real eigenvalues of A with their algebraic multiplicities.

b. Find an eigenvector corresponding to the eigenvalue $\lambda = 1$

6. (30 points) Let T from $\mathbf{R}^2$ to $\mathbf{R}^2$ be the orthogonal projection onto the line spanned by $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

a. Find the matrix of T with respect to the basis $\begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \end{bmatrix}$.

b. Use your answer in part (a) to find the standard matrix of T.