1. (20 points) Suppose that $f$ is differentiable on $\mathbb{R}$.
   
   a. If $f'(x) = 0$ for all $x \in \mathbb{R}$, prove that $f(x) = f(0)$ for all $x \in \mathbb{R}$
   
   b. If $f(0) = 1$ and $|f'(x)| \leq 1$ for all $x \in \mathbb{R}$, prove that $|f'(x)| \leq |x| + 1$ for all $x \in \mathbb{R}$

2. (20 points) Evaluate the following limits
   
   a. $\lim_{x \to \infty} \sqrt{x + \sqrt{x} + \sqrt{x} - \sqrt{x}}$
   
   b. $\lim_{x \to 0^+} x \log x$
   
   c. $\lim_{x \to \infty} x^2 \left( e^{\frac{1}{x}} - e^{\frac{x}{x+1}} \right)$
   
   d. $\lim_{x \to a} \frac{x^\frac{1}{n} - a^\frac{1}{n}}{x - a}$; where $a > 0$ and $n \in \mathbb{N}$

3. (30 points) Assume that $f$ is a continuously differentiable function on $\mathbb{R}$.
   
   a. Let $a \in \mathbb{R}$ and $h > 0$. Show that
      \[ \exists \mu \in (0, 1) : f(a + h) - 2f(a) + f(a - h) = h \left[ f'(a + \theta h) - f'(a - \theta h) \right] \]
   
   b. Let $c$ be a real number. Find the following limit
      \[ \lim_{h \to 0} \frac{f^2(c + 3h) - f^2(c - h)}{h} \]

4. (30 points) Prove that
   \[ \sin x > x - \frac{x^3}{6}; \forall x \in (0, 2\pi] \]

5. (Bonus Question.) (10 points) Let $f$ and $g$ be real functions. Assume that $f$ is differentiable at $a$ and $g$ is differentiable at $f(a)$. Prove that $g \circ f$ is differentiable at $a$ and
   \[ (g \circ f)'(a) = g'(f(a))f'(a) \]