MATH 350 EXAM #3 Spring 2005

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Textbooks, classnotes, crib sheets, or calculators are not permitted.

1. (20 points) Suppose that f is differentiable on $\mathbf{R}$.

   a. If $f^{'}(x) = 0$ for all $x \in \mathbf{R}$, prove that $f(x) = f(0)$ for all $x \in \mathbf{R}$

   b. If $f(0) = 1$ and $|f^{'}(x)| \leq 1$ for all $x \in \mathbf{R}$, prove that $|f^{'}(x)| \leq |x| + 1$ for all $x \in \mathbf{R}$

2. (20 points) Evaluate the following limits

   a. $\lim\limits_{x \to \infty} \sqrt{x + \sqrt{x + \sqrt{x}}} - \sqrt{x}$

   b. $\lim\limits_{x \to 0^+} x \log x$

   c. $\lim\limits_{x \to \infty} x^2 \left(e^{\frac{1}{x}} - e^{\frac{1}{x+1}} \right)$

   d. $\lim\limits_{x \to a} \dfrac{x^{\frac{1}{n}} - a^{\frac{1}{n}}}{x - a}$; where $a > 0$ and $n \in \mathbf{N}$

3. (30 points) Assume that f is a continuously differentiable function on $\mathbf{R}$.

   a. Let $a \in \mathbf{R}$ and $h > 0$. Show that

   $$\exists\, \mu \in (0\ ,\ 1): \quad f(a+h) - 2f(a) + f(a-h) \;=\; h\left[f^{'}(a+\theta h) - f^{'}(a-\theta h)\right]$$

   b. Let c be a real number. Find the following limit

   $$\lim_{h \to 0} \frac{f^2(c+3h) - f^2(c-h)}{h}$$

4. (30 points) Prove that

   $$\sin x > x - \frac{x^3}{6} \ ; \quad \forall x \in (0\ ,\ 2\pi]$$

5. **(Bonus Question.)**(10 points) Let f and g be real functions. Assume that f is differentiable at a and g is differentible at $f(a)$. Prove that $g \circ f$ is differentiable at a and

   $$(g \circ f)^{'}(a) = g^{'}(f(a))f^{'}(a)$$