MATH 582C EXAM #3 Spring 2005

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Textbooks, classnotes, crib sheets, or calculators are not permitted.

1. (30 points) Let ϕ be a polynomial of degree d.

   a. Give the definition of a Schur polynomial as well as the necessary and sufficient conditions on ϕ to be a Schur polynomial.

   b. Give the definition of a Von Neumann polynomial and a simple Von Neumann polynomial. Then, state the necessary and sufficient condition on ϕ to be a simple Von Neumann polynomial .

2. (30 points) Consider the following one-way wave equation

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = f$$

   and the associated *second-order* accurate scheme

$$\frac{7v_m^{n+1} - 8v_m^n + v_m^{n-1}}{6k} + a\delta_0\left(\frac{2v_m^{n+1} + v_m^n}{3}\right) = f_m^{n+3/2}$$

   where δ_0 is the central (first) difference operator defined by

$$\delta_0 v_m^n = \frac{v_{m+1}^n - v_{m-1}^n}{2h}$$

   a. Show that the amplification polynomial corresponding to this scheme is

$$\phi(z) = (7 + 4\mathrm{i}\beta)z^2 - (8 - 2\mathrm{i}\beta)z + 1$$

   where $\beta = a\lambda\sin\theta$ with $\lambda = \frac{k}{h}$, and $\theta = h\xi$.

   b. Analyze the stability of this *two-steps* scheme.

3. (40 points) Consider the following one-way wave equation

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = f$$

   and the associated *Crank-Nicolson* scheme

$$\frac{v_m^{n+1} - v_m^n}{k} + a\frac{v_{m+1}^{n+1} - v_{m-1}^{n+1} + v_{m+1}^n - v_{m-1}^n}{4h} = \frac{f_m^{n+1} + f_m^n}{2}$$

   a. Find the order of accuracy of this scheme.

   b. Analyze the stability of Crank-Nicolson scheme.