1. (20 points) Consider the linear system
\[
\begin{bmatrix}
2 & 0 & -1 \\
-2 & -10 & 0 \\
-1 & -1 & 4
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} =
\begin{bmatrix}
1 \\
-12 \\
2
\end{bmatrix}
\]
Using the starting vector \(x^{(0)} = (0, 0, 0)^T\), carry out two iterations of the Jacobi method.

2. (30 points) Using \(Q\) as in the Gauss-Seidel method, prove that if \(A\) (an \(n \times n\) matrix) is strictly diagonally dominant, then \(||I - Q^{-1}A||_\infty < 1\). Conclude!

3. (20 points) Count the number of multiplications and/or divisions involved in carrying out \(m\) steps of the power method.

4. (30 points) Let \(A \in \mathbb{R}^{n \times n}\) be symmetric and positive definite, and \(b \in \mathbb{R}^n\). Show that \(x^*\) is the solution of the linear system
\[
Ax = b
\]
if and only if \(x^*\) satisfies
\[
J(x^*) \leq J(x) ; \quad \forall x \in \mathbb{R}^n
\]
where
\[
J(x) = \frac{1}{2} x^t Ax - x^t b
\]

5. **Bonus Questions** (40 points)

a. Show that the QR factorization of an invertible matrix \(A \in \mathbb{R}^{n \times n}\) is unique.

b. Let the eigenvalues of \(A\) satisfy \(\lambda_1 > \lambda_2 > ... > \lambda_n\) (not necessarily positive). What value of the parameter \(\mu\) should be used in order for the power method to converge most rapidly to \(\lambda_1\) when applied to \(A + \mu I\)?