MATH 581 EXAM #3 Fall 2003

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Textbooks, classnotes, crib sheets, or calculators are not permitted.

1. (20 points) Consider the linear system

$$\begin{bmatrix} 2 & 0 & -1 \\ -2 & -10 & 0 \\ -1 & -1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -12 \\ 2 \end{bmatrix}$$

   Using the starting vector $x^{(0)} = (0,0,0)^T$, carry out two iterations of the Jacobi method.

2. (30 points) Using Q as in the Gauss-Seidel method, prove that if A ($n \times n$ matrix) is strictly diagonally dominant, then $||I - Q^{-1}A||_\infty < 1$. Conclude!

3. (20 points) Count the number of multiplications and/or divisions involved in carrying out m steps of the power method.

4. (30 points) Let $A \in \mathbf{R}^{n\times n}$ be symmetric and positive definite, and $b \in \mathbf{R}^n$. Show that x^* is the solution of the linear system

$$Ax = b$$

   if and only if x^* satisfies

$$J(x^*) \leq J(x) \quad ; \quad \forall x \in \mathbf{R}^n$$

   where

$$J(x) = \frac{1}{2} x^t A x - x^t b$$

5. **Bonus Questions**(40 points)

   a. Show that the QR factorization of an invertible matrix $A \in \mathbf{R}^{n\times n}$ is unique.

   b. Let the eigenvalues of A satisfy $\lambda_1 > \lambda_2 > ... > \lambda_n$ (not necessarily positive). What value of the parameter μ should be used in order for the power method to converge most rapidly to λ_1 when applied to $A + \mu I$?