MATH 582C EXAM #2 Spring 2005

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Textbooks, classnotes, crib sheets, or calculators are not permitted.

1. (30 points) Use the definition to show that schemes of the form

$$v_m^{n+1} = \alpha v_{m+1}^n + \beta v_{m-1}^n$$

are stable if $|\alpha| + |\beta| \leq 1$.

2. (30 points) Let f be an integrable function in $\mathbf{R}$. We define $\mathcal{L}(f)(s) = F(s)$, the Laplace transform of f, as follows

$$F(s) = \int_0^{+\infty} f(t)\, e^{-ts}\, dt$$

   a. Prove that $\mathcal{L}\left(H(t-a)f(t-a)\right)(s) = e^{-as}F(s)$; where H is the Heaviside step function and a is a real number.

   b. Use Laplace transform to solve the following boundary value problem

$$\text{(BVP)} \quad \begin{cases} \dfrac{\partial u}{\partial t} - \dfrac{\partial^2 u}{\partial x^2} = 0\ ; & x > 0\ ,\ t > 0 \\ u(0,t) = 100H(t-2)\ ; & t > 0 \\ u(x,0) = 0\ ; & x > 0 \end{cases} \tag{1}$$

   You may use the following property to simplify the expression of u.

$$\mathcal{L}\left(\frac{x}{2c\sqrt{\pi}t^{3/2}} e^{-\dfrac{x^2}{4c^2 t}}\right)(s) = e^{-\dfrac{x}{c}\sqrt{s}}$$

   where c is a positive number.

3. (40 points) Consider the following one-way wave equation

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = f \tag{2}$$

and the associated *forward time–central space* (FT-CS) scheme

$$\frac{v_m^{n+1} - v_m^n}{k} + a\frac{v_{m+1}^n - v_{m-1}^n}{k} = f_m^n \tag{3}$$

Is the FT-CS scheme given by Eq.2 a convergent scheme?