MATH 581 EXAM #2 Fall 2003

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Textbooks, classnotes, crib sheets, or calculators are not permitted.

1. (20 points) Solve the following linear systems twice. First, use Gaussian elimination and give the factorization $A = LU$. Second, use Gaussian elimination with scaled row pivoting and determine the factorization of the form $PA = LU$

$$\begin{bmatrix} -1 & 1 & -4 \\ 2 & 2 & 0 \\ 3 & 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ \frac{1}{2} \end{bmatrix}$$

2. (15 points) Let $x = (x_1, x_2, ..., x_n) \in \mathbf{R}^n$. Determine whether these expressions define norms on $\mathbf{R}^n$

   a. $\max |x_2|, |x_3|, ..., |x_n|$

   b. $\sum_{i=1}^{n} |x_i|^3$

   c. $\left\{ \sum_{i=1}^{n} |x_i|^{\frac{1}{2}} \right\}^2$

3. (20 points)

   a. Prove that if A is a symmetric matrix whose leading principal minors are nonsingular, then A has a factorization LDL^T in which L is a unit lower triangular matrix and D is diagonal.

   b. Prove that if A is invertible and has an LU decomposition, then all principal minors of A are nonsingular.

HEY, THERE'S MORE—TURN THE PAGE OVER!

4. (30 points) Consider A and $B \in \mathbf{R}^{n\times n}$. We assume that A is invertible and $||B - A|| < ||A^{-1}||^{-1}$ where $||\ ||$ is a matrix norm subordinate to a vector norm. Prove that:

   a. B is invertible

   b. $B^{-1} = A^{-1}\sum_{k=0}^{\infty}\left(I - BA^{-1}\right)^k$

   c. $||A^{-1} - B^{-1}|| \leq ||A^{-1}||\dfrac{||I - A^{-1}B||}{1 - ||I - A^{-1}B||}$

5. (15 points) Count the number of multiplications and/or divisions needed to invert a unit $n \times n$ lower triangular matrix.