1. (10 points) Consider the subspace $L$ spanned by the vector \[
\begin{bmatrix}
1 \\
2 \\
3 \\
4 \\
5
\end{bmatrix}
\] in $\mathbb{R}^5$. Find a basis of the orthogonal complement $L^\perp$ of $L$.

2. (30 points) State whether each of the following statements are TRUE or FALSE. You do not need to show your work.

a. If $A$ and $B$ are two invertible matrices, then $(A^{-1}B)^{-1} = AB^{-1}$.

b. If the kernel of a matrix $A$ consists of the zero vector only, then the column vectors of $A$ must be linearly independent.

c. If $V$ and $W$ are subspaces of $\mathbb{R}^n$, then their union $V \cup W$ must be a subspace of $\mathbb{R}^n$ as well.

d. If $AB = 0$ for $2 \times 2$ matrices $A$ and $B$, then $BA$ must be the zero matrix as well.

e. If vectors $\vec{v}_1, \vec{v}_2, ..., \vec{v}_m$ span $\mathbb{R}^n$, then $m$ must be equal to $n$.

f. If $A^2 = A$ for an invertible $n \times n$ matrix $A$, then $A$ must be $I_n$.

g. If vectors $\vec{v}_1, \vec{v}_2, ..., \vec{v}_m$ are linearly independent in $\mathbb{R}^n$, then $m$ must be equal to $n$.

h. If $A$ and $B$ are two invertible matrices, then $(A + B)^{-1} = A^{-1} + B^{-1}$.

i. If vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4$ are linearly independent, then $\vec{v}_1, \vec{v}_2, \vec{v}_3$ must be linearly independent.

j. If $A$ is $n \times m$ matrix and $B$ is $p \times m$ matrix, then $AB$ is $n \times p$ matrix.

3. (30 points) Consider a linear transformation $T$ from $\mathbb{R}^2$ to $\mathbb{R}^2$. We are told that the matrix of $T$ with respect to the basis \[
\begin{bmatrix}
3 \\
5
\end{bmatrix}, \begin{bmatrix}
5 \\
8
\end{bmatrix}
\] is \[
\begin{bmatrix}
1 & 9 \\
9 & 7
\end{bmatrix}
\].

Find the standard matrix of $T$.

HEY, THERE’S MORE—TURN THE PAGE OVER!
4. (30 points) Consider the matrix

\[ A = \begin{bmatrix} 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix} \]

a. Find a basis of the kernel of \( A \), and thus determine the dimension of \( \ker A \).

b. Use your answer in part (a) to find \( \text{rank}(A) \), and then determine a basis of \( \text{im}(A) \).