MATH 262 EXAM #2 Springl 2004

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Textbooks, classnotes, crib sheets, or calculators are not permitted.

1. (10 points) Consider the subspace L spanned by the vector $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$ in $\mathbf{R}^5$. Find a basis of the orthogonal complement $L^\perp$ of L.

2. (30 points) State whether each of the following statements are TRUE or FALSE. You do not need to show your work.

   a. If A and B are two invertible matrices, then $(A^{-1}B)^{-1} = AB^{-1}$.
   b. If the kernel of a matrix A consists of the zero vector only, then the column vectors of A must be linearly independent.
   c. If V and W are subspaces of $\mathbf{R}^n$, then their union $V \bigcup W$ must be a subspace of $\mathbf{R}^n$ as well.
   d. If $AB = 0$ for 2×2 matrices A and B, then BA must be the zero matrix as well.
   e. If vectors $\vec{v_1}, \vec{v_2}, ..., \vec{v_m}$ span $\mathbf{R}^n$, then m must be equal to n.
   f. If $A^2 = A$ for an invertible $n \times n$ matrix A, then A must be I_n.
   g. If vectors $\vec{v_1}, \vec{v_2}, ..., \vec{v_m}$ are linearly independent in $\mathbf{R}^n$, then m must be equal to n.
   h. If A and B are two invertible matrices, then $(A+B)^{-1} = A^{-1} + B^{-1}$
   i. If vectors $\vec{v_1}, \vec{v_2}, \vec{v_3}, \vec{v_4}$ are linearly independent, then $\vec{v_1}, \vec{v_2}, \vec{v_3}$ must be linearly independent
   j. If A is $n \times m$ matrix and B is $p \times m$ matrix, then AB is $n \times p$ matrix.

3. (30 points) Consider a linear transformation T from $\mathbf{R}^2$ to $\mathbf{R}^2$. We are told that the matrix of T with respect to the basis $\begin{bmatrix} 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 5 \\ 8 \end{bmatrix}$ is $\begin{bmatrix} 1 & 9 \\ 9 & 7 \end{bmatrix}$.

   Find the standard matrix of T.

HEY, THERE'S MORE—TURN THE PAGE OVER!

4. (30 points) Consider the matrix

$$A = \begin{bmatrix} 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

a. Find a basis of the kernel of A, and thus determine the dimension of $kerA$.

b. Use your answer in part (a) to find $rank(A)$, and then determine a basis of $im(A)$.