MATH 262 EXAM #2 Fall 2005

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section, and (4) a grading table. You must work all the problems on the exam. Show ALL your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Textbooks, classnotes, crib sheets, or calculators are not permitted.

1. (10 points) Consider the subspace L spanned by the vector $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$ in $\mathbf{R}^5$. Find a basis of the orthogonal complement $L^\perp$ of L.

2. (20 points) Consider two subspaces V and W in $\mathbf{R}^n$. Prove that their intersection $V \bigcap W$ must be a subspace of $\mathbf{R}^n$ as well.

3. (30 points) State whether each of the following statements are TRUE or FALSE. You do not need to show your work.

   a. The image of a $m \times n$ matrix is a subspace of $\mathbf{R}^n$

   b. If V and W are subspaces of $\mathbf{R}^n$, then their union $V \bigcup W$ must be a subspace of $\mathbf{R}^n$ as well.

   c. If A is an $m \times n$ matrix, then $dim(KerA) + rank(A) \ = \ m$

   d. If vectors $\vec{v_1}, \vec{v_2}, ..., \vec{v_m}$ span $\mathbf{R}^n$, then m must be equal to n.

   e. If A and B are $m \times n$ matrices, and $\vec{v}$ is in the kernel of both A and B, then $\vec{v}$ must be in the kernel of $A + B$ as well.

   f. If vectors $\vec{v_1}, \vec{v_2}, ..., \vec{v_m}$ are linearly independent in $\mathbf{R}^n$, then m must be equal to n.

4. (20 points) Consider the matrix

$$A \ = \ \begin{bmatrix} 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

   a. Find a basis of the kernel of A, and thus determine the dimension of $kerA$.

   b. Use your answer in part (a) to find $rank(A)$, and then determine a basis of $im(A)$.

5. (20 points) Consider a linear transformation T from $\mathbf{R}^2$ to $\mathbf{R}^2$. We are told that the matrix of T with respect to the basis $\begin{bmatrix} 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 5 \\ 8 \end{bmatrix}$ is $\begin{bmatrix} 1 & 9 \\ 9 & 7 \end{bmatrix}$.

   Find the standard matrix of T.