ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Textbooks, classnotes, crib sheets, or calculators are not permitted.

1. (10 points) Find a nontrivial relation among the following vectors

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

2. (30 points) Let T from $\mathbf{R}^2$ to $\mathbf{R}^2$ be the orthogonal projection onto the line spanned by $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

   a. Find the matrix of T with respect to the basis $\begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \end{bmatrix}$.

   b. Use your answer in part (a) to find the standard matrix of T.

3. (30 points) State whether each of the following statements are TRUE or FALSE. You do not need to show your work.

   a. The image of a $m \times n$ matrix is a subspace of $\mathbf{R}^n$

   b. If V and W are subspaces of $\mathbf{R}^n$, then their union $V \bigcup W$ must be a subspace of $\mathbf{R}^n$ as well.

   c. If A is an $m \times n$ matrix, then $dim(KerA) + rank(A) \ = \ m$

   d. If vectors $\vec{v_1}, \vec{v_2}, ..., \vec{v_m}$ span $\mathbf{R}^n$, then m must be equal to n.

   e. If A and B are $m \times n$ matrices, and $\vec{v}$ is in the kernel of both A and B, then $\vec{v}$ must be in the kernel of $A + B$ as well.

   f. If vectors $\vec{v_1}, \vec{v_2}, ..., \vec{v_m}$ are linearly independent in $\mathbf{R}^n$, then m must be equal to n.

HEY, THERE'S MORE—TURN THE PAGE OVER!

4. (20 points) Consider the matrix

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 2 & 3 \\ 1 & 1 & 3 \end{bmatrix}$$

a. Find a basis of the kernel of A, and thus determine the dimension of $ker A$.

b. Use your answer in part (a) to find $rank(A)$, and thus determine a basis of $im(A)$.

5. (10 points) Find a basis of the subspace of $\mathbf{R}^4$ defined by the equation

$$2x_1 - x_2 + 2x_3 + 4x_4 = 0$$