ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Textbooks, classnotes, crib sheets, or calculators are not permitted.

1. (20 points) Find the limit of each of the following sequences. If a limit does not exist, write "**DNE**" for your answer.

   a. $x_n = \dfrac{\sin((n^4+n+1)/(n^2+1))}{n}$

   b. $y_n = \dfrac{1}{\sqrt{2n^2+1}-n}$

   c. $z_n = \dfrac{n}{2^n}$

   d. $t_n = (-3)^{\frac{1}{2n-1}}$

2. (20 points) Let (x_n) and (y_n) be two convergent sequences. Suppose that $y_n \neq 0$ and $\lim\limits_{n\to+\infty} y_n \neq 0$. Prove that

$$\lim_{n\to+\infty}\left(\frac{x_n}{y_n}\right) = \frac{\lim\limits_{n\to+\infty} x_n}{\lim\limits_{n\to+\infty} y_n}$$

3. (20 points) State whether each of the following statements are **TRUE** or **FALSE**. You do need to show your work when your answer is **FALSE** only.

   a. Any subsequence of a bounded sequence *must* be bounded.

   b. Any convergent sequence *must* be monotone.

   c. Assume (x_n) is a monotone sequence. Then, (x_n) converges *iff* (x_n) is bounded.

   d. Any bounded sequence has *at most* one convergent subsequence.

   e. If $\{I_n\}_{n\in\mathbf{N}}$ is a sequence of *nonempty closed* and *bounded* intervals, then $E = \bigcap\limits_{n\in\mathbf{N}} I_n$ contains at least one number.

HEY, THERE'S MORE—TURN THE PAGE OVER!

4. (20 points) Let (x_n) be a sequence defined by

$$\begin{cases} x_1 = \frac{1}{2} \\ x_{n+1} = x^2{}_n + \frac{3}{16} \end{cases}$$

a. Show that $x_n \geq 0$; $\forall n \in \mathbf{N}$

b. Show that (x_n) is a decreasing sequence.

c. Deduce that (x_n) is a convergent sequence and find its limit.

5. (20 points) Suppose that $x \in \mathbf{R}$, $x_n > 0$, and $\lim_{n\to\infty} x_n = x$. Prove that

$$\lim_{n\to\infty} \sqrt{x_n} = \sqrt{x}$$

6. **Bonus Questions**(10 points) Let (x_n) and (y_n) be two sequences such that $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$. Consider the sequence (z_n) defined by

$$z_n = \inf(x_n, y_n) \ ; \quad \forall n \in \mathbf{N}$$

Prove that (z_n) is a convergent sequence and find its limit.