1. (20 points) Find the polynomial \( f(t) \) of degree 3 such that \( f(1) = 1 \), \( f(2) = 5 \), \( f'(1) = 2 \), and \( f'(2) = 9 \), where \( f'(t) \) is the derivative of \( f(t) \). Graph this polynomial.

2. (20 points) Consider the transformation \( T \) from \( \mathbb{R}^2 \) to \( \mathbb{R}^3 \) given by

\[
T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}
\]

Is this transformation linear? If so, find its matrix.

3. (15 points) Consider the linear system

\[
\begin{aligned}
    x + y - z &= -2 \\
    3x - 5y + 13z &= 18 \\
    x - 2y + 5z &= k
\end{aligned}
\]

where \( k \) is an arbitrary constant.

i. For which value(s) of \( k \), does this system have one or infinitely many solutions?

ii. For each value of \( k \) you found in part (i), how many solutions does this system have?

iii. Find all the solutions for each value of \( k \).

4. (20 points) For which values of the constant \( k \) is the following matrix invertible?

\[
A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & k \\ 1 & 4 & k^2 \end{bmatrix}
\]

Find the rank of the matrix \( A \).

HEY, THERE’S MORE—TURN THE PAGE OVER!
5. (10 points) State whether each of the following statements are TRUE or FALSE. You do not need to show your work.

a. The matrix \( A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \) is in reduced row-echelon form (rref).

b. A system of five equations in three unknowns is always inconsistent.

c. If \( A \) is a \( 3 \times 4 \) matrix and \( \vec{v} \) is a vector in \( \mathbb{R}^3 \), then the product \( A\vec{v} \) is a vector in \( \mathbb{R}^3 \).

d. There is a \( 3 \times 4 \) matrix with rank 4.

e. The inverse of an \( n \times m \) matrix is an \( m \times n \) matrix.

f. Let \( A \) and \( B \) be two invertible matrices. Then,
   i. \((A + B)^2 = A^2 + 2AB + B^2\)
   ii. \((ABA^{-1})^3 = AB^3A^{-1}\)
   iii. \(ABA^{-1} = B\)
   iv. \((A^{-1}B)^{-1} = B^{-1}A\)
   v. \((A + B)^{-1} = A^{-1} + B^{-1}\)

6. (15 points) If possible, compute the following matrix products.

a. \( \begin{bmatrix} 0 & 0 & 1 \\ \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \)

b. \( \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \)

c. \( \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \)

d. \( \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \)

e. \( \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \)