1. (30 points) Consider the linear system

\[
\begin{align*}
  x + y - z &= 2 \\
  x + 2y + z &= 3 \\
  x + y + (k^2 - 5)z &= k
\end{align*}
\]

where \( k \) is an arbitrary constant.

a. For which value(s) of \( k \) this system is inconsistent?

b. For which value(s) of \( k \) does this system have one solution? Find the solution.

c. For which value(s) of \( k \) does this system have infinitely many solutions? Find all the solutions.

2. (10 points) Consider two (nonzero) perpendicular vectors \( \vec{u} \), and \( \vec{w} \) in \( \mathbb{R}^2 \). Show that the transformation

\[ T(\vec{x}) = \vec{x} + (\vec{u} \cdot \vec{x})\vec{w} \]

is a shear parallel to the line \( L \) spanned by \( \vec{w} \).

3. (15 points) State whether each of the following statements are TRUE or FALSE. You do not need to show your work.

a. The matrix \( A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \) is in reduced row-echelon form (rref).

b. A system of four equations in three unknowns is always inconsistent.

c. If \( A \) is a \( 3 \times 4 \) matrix and \( \vec{v} \) is a vector in \( \mathbb{R}^4 \), then the product \( A\vec{v} \) is a vector in \( \mathbb{R}^4 \).

d. There is a \( 3 \times 4 \) matrix with rank 4.

e. If \( \vec{u} \), \( \vec{v} \), and \( \vec{w} \) are non zero vectors in \( \mathbb{R}^2 \), then \( \vec{w} \) must be a linear combination of \( \vec{u} \) and \( \vec{v} \).

HEY, THERE’S MORE—TURN THE PAGE OVER!
4. (15 points) How many types of $2 \times 3$ matrices in reduced row-echelon form are there?

5. (30 points) Let $L$ be the line in $\mathbb{R}^3$ that consists of all scalar multiples of \[ \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \] and $T$ a transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ such that

\[ T(\vec{x}) = 2(\text{proj}_L \vec{x}) - \vec{x} \]

where $\text{proj}_L \vec{x}$ is the orthogonal projection of $\vec{x}$ onto $L$.

a. Show that $T$ is a linear transformation.
b. Find the matrix corresponding to $T$.
c. Use question (b) to find $T(\vec{v})$ for $\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. 