Physics 100A Homework 8 – Chapter 9

9.4 Two air-track carts move toward one another on an air track. Cart 1 has a mass of 0.35 kg and a speed of 1.2 m/s. Cart 2 has a mass of 0.61 kg.

A)What speed must cart 2 have if the total momentum of the system is to be zero?

B)Since the momentum of the system is zero, does it follow that the kinetic energy of the system is also zero?

4. **Picture the Problem**: The two carts approach each other on a frictionless track at different speeds.

Strategy: Add the momenta of the two carts and set it equal to zero. Solve the resulting expression for v_2. Then use equation 7-6 to find the total kinetic energy of the two-cart system. Let cart 1 travel in the positive direction.

Solution: 1. (a) Set $\sum \vec{\mathbf{p}} = 0$ and solve for v_2:

$$\sum \vec{\mathbf{p}} = m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2 = 0$$

This is a one-dimensional problem so the arrow is dropped but the direction is taken into consideration via the plus/minus sign.

$$v_2 = \left| -\frac{m_1 v_1}{m_2} \right| = \frac{(0.35\text{ kg})(1.2\text{ m/s})}{0.61\text{ kg}} = \boxed{0.69\text{ m/s}}$$

The absolute of the quantity is used to calculate the speed, which is a scalar.

2. (b) $\boxed{\text{No}}$, kinetic energy is always greater than or equal to zero.

3. (c) Use equation 7-6 to sum the kinetic energies of the two carts:

$$\sum K = \tfrac{1}{2}m_1 v_1^{\,2} + \tfrac{1}{2}m_2 v_2^{\,2}$$
$$= \tfrac{1}{2}(0.35\text{ kg})(1.2\text{ m/s})^2 + \tfrac{1}{2}(0.61\text{ kg})(0.69\text{ m/s})^2$$
$$= \boxed{0.40\text{ J}}$$

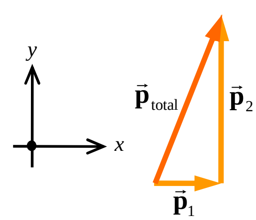
Insight: If cart 1 is traveling in the positive $\hat{\mathbf{x}}$ direction, then its momentum is $(0.42\text{ kg}\cdot\text{m/s})\hat{\mathbf{x}}$ and the momentum of cart 2 is $(-0.42\text{ kg}\cdot\text{m/s})\hat{\mathbf{x}}$.

9.7 Object 1 has a mass m_1 and a velocity $\vec{v}_1 = 2.20\text{ m/s}\,\hat{x}$.

Object 2 has a mass m_2 and a velocity $\vec{v}_2 = 3.50\text{ m/s}\,\hat{y}$.

The total momentum of these two objects has a magnitude of 17.6 kg ·m/s and points in a direction 66.5° above the positive x axis.

7. **Picture the Problem**: The individual momenta and final momentum vectors are depicted at right.



Strategy: The momenta of the two objects are perpendicular. Because of this we can say that the momentum of object 1 is equal to the *x*-component of the total momentum and the momentum of object 2 is equal to the *y*-component of the total momentum. Find the momenta of objects 1 and 2 in this manner and divide by their speeds to determine the masses.

Solution: 1. Find $p_{\text{total},\,x}$ and divide by v_1:

$$p_1 = m_1 v_1 = p_{\text{total},\,x} = p_{\text{total}}\cos\theta = (17.6\text{ kg}\cdot\text{m/s})(\cos 66.5°) = \underline{\underline{7.02\text{ kg}\cdot\text{m}}}$$

$$m_1 = \frac{p_1}{v_1} = \frac{7.02\text{ kg}\cdot\text{m/s}}{2.80\text{ m/s}} = \boxed{2.51\text{ kg}}$$

2. Find $p_{\text{total},\,y}$ and divide by v_2:

$$p_2 = m_2 v_2 = p_{\text{total, y}} = p_{\text{total}}\sin\theta = (17.6\text{ kg}\cdot\text{m/s})(\sin 66.5°) = \underline{\underline{16.1\text{ kg}\cdot\text{m/s}}}$$

$$m_2 = \frac{p_2}{v_2} = \frac{16.1\text{ kg}\cdot\text{m/s}}{3.10\text{ m/s}} = \boxed{5.21\text{ kg}}$$

Insight: Note that object 2 has the larger momentum because the total momentum points mostly in the $\hat{\mathbf{y}}$ direction. The two objects have similar speeds, so object 2 must have the larger mass in order to have the larger momentum.

Impulse on a Ball

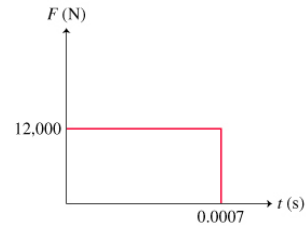
In a baseball game the batter swings and gets a good solid hit. His swing applies a force of 12,000 N to the ball for a time of 0.7×10^{-3} s.

A) Assuming that this force is constant, what is the magnitude J of the impulse on the ball?

$J = F\Delta t = (12{,}000)(0.7\times10^{-3}) = 8.4$ **kg ·m/s**

B) The net force versus time graph has a rectangular shape. Often in physics geometric properties of graphs have physical meaning.

For this graphs the area of the rectangle corresponds to the impulse.



C) If both the graph representing the constant net force and the graph representing the variable net force represent the same impulse acting on the baseball, the two graphs must have the same area.

D) Assume that a pitcher throws a baseball so that it travels in a straight line parallel to the ground. The batter then hits the ball so it goes directly back to the pitcher along the same straight line. Define the direction the pitcher originally throws the ball as the +*x* direction.

The impulse on the ball caused by the bat will be in the negative x-direction.

$$\vec{J} = \vec{p}_f - \vec{p}_i$$

$$\vec{J} = m_{ball}\vec{v}_f(-\hat{x}) - m_{ball}\vec{v}_i\hat{x}$$

$$\vec{J} = -m_{ball}(v_f + v_i)\hat{x}$$

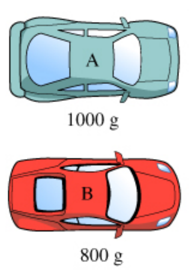
E) Now assume that the pitcher in Part D throws a 0.145-kg baseball parallel to the ground with a speed of 32 m/s in the +*x* direction. The batter then hits the ball so it goes directly back to the pitcher along the same straight line. What is the ball's velocity just after leaving the bat if the bat applies an impulse of -8.4 Ns to the baseball?

$$\frac{J}{m_{ball}} = -v_f - v_i$$

$$v_f = --v_i - \frac{J}{m_{ball}} = -32 - \frac{(-84)}{0.145} = -26.0 \text{ m/s} \qquad \text{(to the left)}$$

Kinetic Energy and Momentum

The two toy cars shown in the figure, with masses as given in the figure, are ready to race. Both cars begin from rest. For each question, state whether the correct answer is car A, car B, or whether the two cars have equal values for the parameter in question.



For the next three parts assume that the cars' motors supply the same force to each car over the course of a 1.0-meter race.

A)Which car crosses the finish line 1.0 m away first? Car B wins.

$p_{initial} = 0$ for both cars

Then $F = ma$. Since the force is the same the car with the smallest mass will have the larger acceleration and will cover the 1.0 m distance faster.

B) Which car has the larger kinetic energy when it crosses the finish line 1.0 m away? The same.

$W = Fd = \Delta K = K_{final} - K_{initial} = K_{final}$ Since the force and displacement are the same the cars will have the same final kinetic energy.

C) Which car has a larger momentum when it crosses the finish line 1.0 m away?

From the kinetic energy $v_{final} = \sqrt{(2W)/m}$

And $p_{final} = mv_{final} = \sqrt{2Wm}$ Since the work is the same the car with the largest mass will have the larger momentum.

D) Which car has traveled farther after 10s? Car B with the larger acceleration.

E) After 10 s which car has a larger kinetic energy? Car B would have travelled the largest distance and therefore experienced the largest work which corresponds to a higher final kinetic energy.

F) After 10 s which car has a larger momentum?

$p_{final} = F \Delta t$ If the force and the time are the same, the final momentum is the same for both.

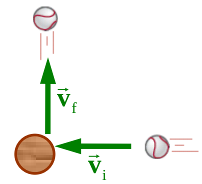
9.19) A 0.14-kg baseball moves toward home plate with a velocity $\vec{v}_i = (-36\text{ m/s})\hat{x}$

After striking the bat, the ball moves vertically upward with a velocity $\vec{v}_f = (18\text{ m/s})\hat{y}$.

A) Find the direction of the impulse delivered to the ball by the bat. Assume that the ball and bat are in contact for 1.5ms.

B)Find the magnitude of the impulse delivered to the ball by the bat. Assume that the ball and bat are in contact for 1.5ms.

19. **Picture the Problem**: The ball rebounds from the bat in the manner indicated by the figure at right.



Strategy: The impulse is equal to the vector change in the momentum. Analyze the *x* and *y* components of $\Delta\vec{\mathbf{p}}$ separately, then use the components to find the direction and magnitude of $\vec{\mathbf{I}}$.

Solution: 1. (a) Find Δp_x: $\Delta p_x = m(v_{fx} - v_{ix}) = (0.14\text{ kg})[0-(-36)\text{ m/s}] = \underline{\underline{5.0\text{ kg}\cdot\text{m/s}}}$

2. Find Δp_y: $\Delta p_y = m(v_{fy} - v_{iy}) = (0.14\text{ kg})(18-0\text{ m/s}) = \underline{\underline{2.5\text{ kg}\cdot\text{m/s}}}$

3. Use equation 9-6 to find $\vec{\mathbf{I}}$: $\vec{\mathbf{I}} = \Delta\vec{\mathbf{p}} = (5.0\text{ kg}\cdot\text{m/s})\hat{\mathbf{x}} + (2.5\text{ kg}\cdot\text{m/s})\hat{\mathbf{y}}$

4. Find the direction of $\vec{\mathbf{I}}$: $\theta = \tan^{-1}\left(\frac{I_y}{I_x}\right) = \tan^{-1}\left(\frac{2.5}{5.0}\right) = \boxed{27°}$ above the horizontal

5. Find the magnitude of $\vec{\mathbf{I}}$: $I = \sqrt{I_x^2 + I_y^2} = \sqrt{(5.0\text{ kg}\cdot\text{m/s})^2 + (2.5\text{ kg}\cdot\text{m/s})^2} = \boxed{5.6\text{ kg}\cdot\text{m/s}}$

6. (b) If the mass of the ball were doubled the impulse would double in magnitude. There would be no change in the direction.

7. (c) If $\Delta\vec{\mathbf{p}}$ of the ball is unchanged, the impulse delivered to the ball would not change, regardless of the mass of the bat.

Insight: The impulse brings the ball to rest horizontally but gives it an initial horizontal speed. Verify for yourself that this ball will travel straight upward 16.5 m (54 feet) before falling back to Earth. An easy popup!

Momentum in an Explosion

A giant "egg" explodes as part of a fireworks display. The egg is at rest before the explosion, and after the explosion, it breaks into two pieces, with the masses indicated in the diagram, traveling in opposite directions

A)What is the momentum $p_{A,i}$ of piece A before the explosion?

Since the egg is initially at rest the initial momentum is zero for both pieces.

B) During the explosion, is the force of piece A on piece B greater than, less than, or equal to the force of piece B on piece A?

By Newton's third law the force of A on B has the same magnitude but opposite direction to the force of B on A.

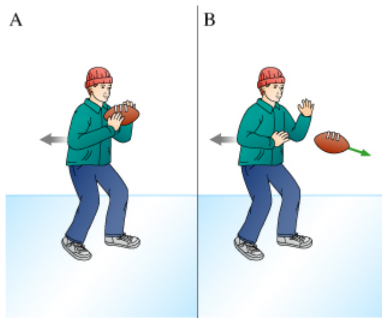
C)The momentum of piece B is measured to be 500 kg·m/s after the explosion. Find the momentum $p_{A,f}$ of piece A after the explosion.

$p_{total,f} = p_{total,i} = 0$

$p_{A,f} + p_{B,f} = 0$ $p_{A,f} = -p_{B,f} = -500$ kg·m/s.

Catching a Ball on Ice

Olaf is standing on a sheet of ice that covers the football stadium parking lot in Buffalo, New York; there is negligible friction between his feet and the ice. A friend throws Olaf a ball of mass 0.400 kg that is traveling horizontally at 11.8 m/s. Olaf's mass is 70.6 kg.



A)If Olaf catches the ball, with what speed v_f do Olaf and the ball move afterward?

$$p_f = p_i$$

$$p_f = p_{O,i} + p_{b,i} \qquad p_f = (m_O + m_b)v_f = p_{b,i}$$ “Stick together”

$$v_f = \frac{m_b v_{b,i}}{(m_O + m_b)} = \frac{(0.4)(11.8)}{(70.6 + 0.4)} = 6.65 \text{ cm/s}$$

B) the ball hits Olaf and bounces off his chest horizontally at 7.10 m/s the opposite direction, what is his speed v_f after the collision?

$$p_{O,f} + p_{b,f} = p_{O,i} + p_{b,i}$$

$$p_{O,f} = p_{b,i} - p_{b,f} = m_b(v_{b,i} - v_{b,f})$$

$$v_{O,f} = \frac{m_b}{m_O}(v_{b,i} - v_{b,f}) = \frac{0.4}{70.6}(11.8 - (-7.1)) = 10.7$$ cm/s, in the original direction of the ball

9.21) Two groups of canoeists meet in the middle of a lake. After a brief visit, a person in canoe 1 pushes on canoe 2 to separate the canoes. Suppose the speeds of the two canoes after they are pushed apart are 0.58 m/s for canoe 1 and 0.42 m/s for canoe 2.

If the mass of canoe 1 is 320 kg, what is the mass of canoe 2?

21. **Picture the Problem**: The two canoes are pushed apart by the force exerted by a passenger.

Strategy: By applying the conservation of momentum we conclude that the total momentum of the two canoes after the push is zero, just as it was before the push. Set the total momentum of the system to zero and solve for m_2. Let the velocity $\vec{\mathbf{v}}_1$ point in the negative direction, $\vec{\mathbf{v}}_2$ in the positive direction.

Solution: Set $p_{\text{total}} = 0$ and solve for m_2:

$$p_{1x} + p_{2x} = 0 = m_1 v_{1x} + m_2 v_{2x}$$

$$m_2 = \frac{-m_1 v_{1x}}{v_{2x}} = \frac{-(320 \text{ kg})(-0.58 \text{ m/s})}{0.42 \text{ m/s}} = \boxed{440 \text{ kg}}$$

Insight: An alternative way to find the mass is to use the equations of kinematics in a manner similar to that described in Example 9-3.

9.25) A 92-kg astronaut and a 1200-kg satellite are at rest relative to the space shuttle. The astronaut pushes on the satellite, giving it a speed of 0.14 m/s directly away from the shuttle. Seven-and-a-half seconds later the astronaut comes into contact with the shuttle.

What was the initial distance from the shuttle to the astronaut?

25. **Picture the Problem**: The astronaut and the satellite move in opposite directions after the astronaut pushes off. The astronaut travels at constant speed a distance *d* before coming in contact with the space shuttle.

Strategy: As long as there is no friction the total momentum of the astronaut and the satellite must remain zero, as it was before the astronaut pushed off. Use the conservation of momentum to determine the speed of the astronaut, and then multiply the speed by the time to find the distance. Assume the satellite's motion is in the negative *x*-direction.

Solution: 1. Find the speed of the astronaut using conservation of momentum:

$$p_a + p_s = 0 = m_a v_a + m_s v_s$$
$$v_a = -\frac{m_s v_s}{m_a}$$

2. Find the distance to the space shuttle:

$$d = v_a t = -\frac{m_s v_s}{m_a} t = -\frac{(1200\text{ kg})(-0.14\text{ m/s})}{(92\text{ kg})}(7.5\text{ s}) = \boxed{14\text{ m}}$$

Insight: One of the tricky things about spacewalking is that whenever you push on a satellite or anything else, because of Newton's Third Law you yourself get pushed ! Conservation of momentum makes it easy to predict your speed.

A Girl in a Trampoline

A girl of mass $m_1 = 60$ kilograms springs from a trampoline with an initial upward velocity of $v_i = 8.0$ meters per second. At height $h = 2.0$ meters above the trampoline, the girl grabs a box of mass $m_2 = 15$ kilograms.

A) What is the speed v_{before} of the girl immediately before she grabs the box?

Here we use kinematics.

$$v_{before}^2 = v_i^2 - 2gy$$

$$v_{before} = \sqrt{(80)^2 - 2(98)(2)} = 4.98 \text{ m/s}$$

B) What is the speed v_{after} of the girl immediately after she grabs the box?

Now we use conservation of momentum.

$$p_f = m_b v_{b,i} + m_g v_{g,before}, \qquad p_f = (m_g + m_b) v_{after}, \qquad v_{b,i} = 0$$

$$v_{after} = \frac{m_g v_{g,before}}{(m_g + m_b)} = \frac{(60)(498)}{(60+15)} = 3.98 \text{ m/s}$$

C) This "collision" is inelastic.

D) What is the maximum height h_{max} that the girl (with box) reaches? Measure h_{max} with respect to the top of the trampoline.

Let us start at the level in which the girl grabs the box and then add the additional height.

$v_f^2 = v_i^2 - 2gy$, the final velocity is zero as the girl and box reach the maximum height.

$$y = \frac{v_i^2}{2g} = \frac{v_{after}^2}{2g} = \frac{(398)^2}{2(98)} = 0.81 \text{ m}$$

And $h_{max} = 0.81 + 20 = 2.81$ m

A One-Dimensional Inelastic Collision

Block 1, of mass m_1 = 8.70 kg, moves along a frictionless air track with speed $v_1 = 15.0$ m/s. It collides with block 2, of mass m_2 = 23.0 kg , which was initially at rest. The blocks stick together after the collision.

A) Find the magnitude p_i of the total initial momentum of the two-block system.

$p_i = m_1 v_{1,i} + m_2 v_{2,i} = (87)(15) + 0$ =131 Kg·m/s

B) Find v_f , the magnitude of the final velocity of the two-block system.

$$p_f = m_{total} v_f = p_i \qquad v_f = \frac{p_i}{m_{total}} = \frac{131}{(8.7+23)} = 4.12 \text{ m/s}$$

C) What is the change $\Delta K = K_f - K_i$ in the system's kinetic energy due to the collision?

$$\Delta K = \frac{1}{2} m_{total} v_f^2 - \frac{1}{2} m_1 v_1^2 - \frac{1}{2} m_2 v_2^2$$

$$\Delta K = \frac{1}{2}(8.7+23)(4.12)^2 - \frac{1}{2}(8.7)(15)^2 - 0 = -710 \text{ J}$$

9.28) A cart of mass m moves with a speed v on a frictionless air track and collides with an identical cart that is stationary.

If the two carts stick together after the collision, what is the final kinetic energy of the system?

$$p_f = m_{total} v_f = p_i = mv \qquad v_f = \frac{mv}{(m+m)} = \frac{v}{2}$$

$$K_f = \frac{1}{2} m_{total} v_f^2 = \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 = 0.25mv^2$$

28. **Picture the Problem**: The two carts collide on a frictionless track and stick together.

Strategy: The collision is completely inelastic because the two carts stick together. Momentum is conserved during the collision because the track has no friction. The two carts move as if they were a single object after the collision. Use the conservation of momentum to find the final speed of the carts and final kinetic energy of the system.

Solution: 1. Conserve momentum to find the final speed of the carts:

$$p_i = p_f$$

$$mv + m(0) = 2mv_f \quad \Rightarrow \quad v_f = \frac{mv}{2m} = \frac{v}{2}$$

2. Use equation 7-6 to find the final kinetic energy:

$$K_f = \tfrac{1}{2}(2m)v_f^{\,2} = m\left(\tfrac{1}{2}v\right)^2 = \boxed{\tfrac{1}{4}mv^2}$$

Insight: Half of the initial kinetic energy is gone, having been converted to heat, sound, and permanent deformation of material during the inelastic collision.

9.37) A 732-kg car stopped at an intersection is rear-ended by a 1720-kg truck moving with a speed of 15.5 m/s.
A) If the car was in neutral and its brakes were off, so that the collision is approximately elastic, find the final speed of the truck.
B) Find the final speed of the car.

Let us derive the equations from the book.

Momentum conservation with $v_{1,i} = v_0$ and $v_{2,i} = 0$

$$m_1 v_{1,f} + m_2 v_{2,f} = m_1 v_0 \qquad \text{Equation 1}$$

$$m_1(v_{1,f} - v_0) = -m_2 v_{2,f} \qquad \text{Equation 2}$$

Kinetic energy conservation

$$\frac{1}{2}m_1 v_{1,f}^2 + \frac{1}{2}m_2 v_{2,f}^2 = \frac{1}{2}m_1 v_0^2$$

$$m_1(v_{1,f}^2 - v_0^2) = -m_2 v_{2,f}^2$$

$$m_1(v_{1,f} - v_0)(v_{1,f} + v_0) = -m_2 v_{2,f}^2 \qquad \text{Equation 3}$$

Substituting from equation 2 into equation 3

$$-m_2 v_{2,f}(v_{1,f} + v_0) = -m_2 v_{2,f}^2$$

$$v_{1,f} + v_0 = v_{2,f} \qquad \text{Equation 4}$$

Substitute into equation 1

$$m_1 v_{1,f} + m_2(v_{1,f} + v_0) = m_1 v_0$$

Collecting terms

$$(m_1 + m_2)v_{1,f} = (m_1 - m_2)v_0$$

$$v_{1,f} = \frac{(m_1 - m_2)}{(m_1 + m_2)} v_0$$

And returning to equation 4

$$v_{2,f} = \frac{2m_1}{(m_1 + m_2)} v_0$$

37. **Picture the Problem**: The truck strikes the car from behind. The collision sends the car lurching forward and slows down the speed of the truck.

Strategy: This is a one-dimensional, elastic collision where one of the objects (the car) is initially at rest. Therefore, equation 9-12 applies and can be used to find the final speeds of the vehicles. Let m_1 be the mass of the truck, m_2 be the mass of the car, and v_0 be the initial speed of the truck.

Solution: 1. Use equation 9-12 to find $v_{1,\mathrm{f}}$:

$$v_{1,\mathrm{f}} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_0 = \left(\frac{1720 - 732\ \mathrm{kg}}{1720 + 732\ \mathrm{kg}}\right)(15.5\ \mathrm{m/s}) = \boxed{6.25\ \mathrm{m/s}} = v_{\mathrm{truck}}$$

2. Use equation 9-12 to find $v_{2,\mathrm{f}}$:

$$v_{2,\mathrm{f}} = \left(\frac{2m_1}{m_1 + m_2}\right) v_0 = \left[\frac{2(1720\ \mathrm{kg})}{1720 + 732\ \mathrm{kg}}\right](15.5\ \mathrm{m/s}) = \boxed{21.7\ \mathrm{m/s}} = v_{\mathrm{car}}$$

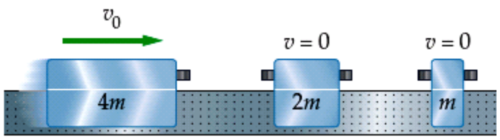
Insight: The elastic collision produces a bigger jolt for the car. If the collision were instead inelastic and the two vehicles stuck together, the final speed of the car (and the truck) would be 10.9 m/s.

9.42) The three air carts shown in the figure have masses, reading from left to right, of 4m, 2m, and m, respectively. The most massive cart has an initial speed of v_0; the other two carts are at rest initially. All carts are equipped with spring bumpers that give elastic collisions.

A) Find the final speed of each cart. (Assume the air track is long enough to accommodate all collisions.)

We use the same equations as in the last problem:

42. **Picture the Problem**: The cart 4*m* collides with the cart 2*m*, which is given kinetic energy as a result and later collides with the cart *m*.



Strategy: In each case a moving cart collides with a cart that is at rest, so application of equation 9-12 will yield the final velocities of all the carts. First apply equation 9-12 to the collision between carts 4*m* and 2*m*, then to the collision between 2*m* and *m*. Let the 4*m* cart be called cart 4, the 2*m* cart be called cart 2, and the *m* cart be called cart 1:

Solution: 1. (a) Apply equation 9-12 to the first collision:

$$v_{4,\text{f}} = \left(\frac{m_4 - m_2}{m_4 + m_2}\right)v_{4,\text{i}} = \left(\frac{4m - 2m}{4m + 2m}\right)v_0 = \boxed{\tfrac{1}{3}v_0}$$

$$v_{2,\text{f}} = \left(\frac{2m_4}{m_4 + m_2}\right)v_{4,\text{i}} = \left[\frac{2(4m)}{4m + 2m}\right]v_0 = \tfrac{4}{3}v_0$$

2. Apply equation 9-12 to the second collision. In this case cart 2 has an initial speed of $\frac{4}{3}v_0$:

$$v_{2,\text{f}} = \left(\frac{m_2 - m_1}{m_2 + m_1}\right)v_{2,\text{i}} = \left(\frac{2m - m}{2m + m}\right)\left(\tfrac{4}{3}v_0\right) = \boxed{\tfrac{4}{9}v_0}$$

$$v_{1,\text{f}} = \left(\frac{2m_2}{m_2 + m_1}\right)v_{2,\text{i}} = \left[\frac{2(2m)}{2m + m}\right]\left(\tfrac{4}{3}v_0\right) = \boxed{\tfrac{16}{9}v_0}$$

3. (b) Verify that $K_\text{i} = K_\text{f}$ by writing using equation 7-6 and dividing both sides by mv_0^2:

$$\tfrac{1}{2}(4m)v_0^2 \stackrel{?}{=} \tfrac{1}{2}(4m)\left(\tfrac{1}{3}v_0\right)^2 + \tfrac{1}{2}(2m)\left(\tfrac{4}{9}v_0\right)^2 + \tfrac{1}{2}(m)\left(\tfrac{16}{9}v_0\right)^2$$

$$2 = \frac{2}{9} + \frac{16}{81} + \frac{256}{162} = \frac{36}{162} + \frac{32}{162} + \frac{256}{162} = \frac{324}{162} = 2$$

Insight: Note that due to the transfer of kinetic energy via collisions, the cart with the smallest mass ends up with the largest speed.