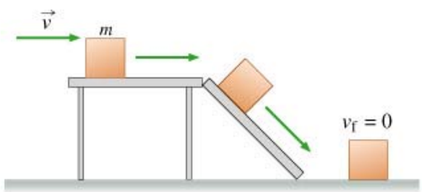
Physics 100A   Homework 7 – Chapter 8

Where's the Energy?

In this problem, we will consider the following situation as depicted in the diagram: A block of mass m slides at a speed v along a horizontal smooth table. It next slides down a smooth ramp, descending a height h, and then slides along a horizontal rough floor, stopping eventually. Assume that the block slides slowly enough so that it does not lose contact with the supporting surfaces (table, ramp, or floor).

You will analyze the motion of the block at different moments using the law of conservation of energy.



A) Which word in the statement of this problem allows you to assume that the table is frictionless?

Smooth
If the problem had friction it would say "rough".

B) Expression of conservation of energy.

Without friction there is no non-conservative work. Energy is conserved:

$$\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f$$

C) As the block slides down the incline
K increases, U decreases, E stays the same

D) Speed at the bottom.

The answer assumes
$v_i = v_{top} = v \quad h_i = h_{top} = h \quad v_f = v_{bottom} \quad h_f = h_{bottom} = 0$

From the conservation of energy equation in part B)

$$v_b = \sqrt{(v^2 + 2gh)}$$

E) From the bottom of the ramp until the block stops.

The potential energy is zero, since in this case the floor has been chosen for U=0.
The initial kinetic energy is diminished due to the action of friction.

Work Energy Theorem

$$W_{nc} = K_f + U_f - K_i - U_i$$
$$W_{nc} = K_f \text{ -} K_i$$
$$K_i + W_{nc} = K_f$$
$$\frac{1}{2}mv_i^2 + W_{nc} = \frac{1}{2}mv_f^2$$

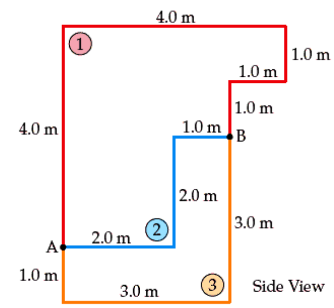
F) As the block slides across the floor
K decreases, U stays the same, E decreases

G) Friction is responsible for the decrease in mechanical energy.

H) Since the block comes to a stop at the level where U=0, the energy lost to friction is
$$E = \frac{1}{2}mv^2 + mgh$$

8.2 Calculate the work done by gravity as a 3.2 kg object is moved from point A to point B in the figure along paths 1, 2, and 3.

8.2
2. **Picture the Problem**: The three paths of the object are depicted at right.



Strategy: Find the work done by gravity $W = mgy$ when the object is moved downward, $W = -mgy$ when it is moved upward, and zero when it is moved horizontally. Sum the work done by gravity for each segment of each path.

Solution: 1. Calculate the work for path 1:

$$W_1 = mg\left[-y_1 + 0 + y_2 + 0 + y_3\right]$$
$$= mg\left[-(4.0\text{ m}) + (1.0\text{ m}) + (1.0\text{ m})\right]$$
$$W_1 = (3.2\text{ kg})(9.81\text{ m/s}^2)(-2.0\text{ m}) = \boxed{-63\text{ J}}$$

2. Calculate W for path 2:

$$W_2 = mg\left[0 - y_4 + 0\right] = (3.2\text{ kg})(9.81\text{ m/s}^2)\left[-2.0\text{ m}\right] = \boxed{-63\text{ J}}$$

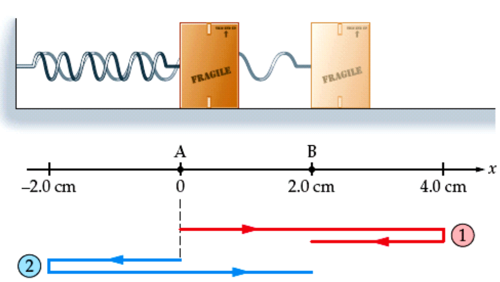
3. Calculate W for path 3:

$$W_3 = mg\left[y_5 + 0 - y_6\right] = (3.2\text{ kg})(9.81\text{ m/s}^2)\left[(1.0\text{ m}) - (3.0\text{ m})\right] = \boxed{-63\text{ J}}$$

Insight: The work is path-independent because gravity is a conservative force.

8.4 A 4.1 kg block is attached to a spring with a force constant of 550 N/m , as shown in the figure . Find the work done by the spring on the block as the block moves from A to B along paths 1 and 2.

4. **Picture the Problem**: The physical situation is depicted at right.



Strategy: Use equation $W = \frac{1}{2}kx^2$ (equation 7-8) to find the work done by the spring, but caution is in order: This work is positive when the force exerted by the spring is in the same direction that the block is traveling, but it is negative when they point in opposite directions. One way to keep track of that sign convention is to say that $W = \frac{1}{2}k\left(x_i^2 - x_f^2\right)$. That way the work will always be negative if you start out at $x_i = 0$ because the spring force will always be in the opposite direction from the stretch or compression.

Solution: 1. (a) Sum the work done by the spring for each segment of path 1:

$$W_1 = \tfrac{1}{2}k\left[\left(x_1^2 - x_2^2\right) + \left(x_2^2 - x_3^2\right)\right]$$
$$= \tfrac{1}{2}(550\text{ N/m})\left\{\left[0^2 - (0.040\text{ m})^2\right] + \left[(0.040\text{ m})^2 - \left(0.020\text{ m}^2\right)\right]\right\}$$
$$W_1 = (-0.44\text{ J}) + (0.33\text{ J}) = \boxed{-0.11\text{ J}}$$

2. Sum the work done by the spring for each segment of path 2:

$$W_2 = \tfrac{1}{2}k\left[\left(x_1^2 - x_4^2\right) + \left(x_4^2 - x_3^2\right)\right]$$
$$= \tfrac{1}{2}(550\text{ N/m})\left\{\left[0^2 - (-0.020\text{ m})^2\right] + \left[(-0.020\text{ m})^2 - \left(0.020\text{ m}^2\right)\right]\right\}$$
$$W_2 = (-0.1\text{ J}) + (0\text{ J}) = \boxed{-0.11\text{ J}}$$

3. (b) The work done by the spring will stay the same if you increase the mass because the results do not depend on the mass of the block.

Insight: The work done by the spring is negative whenever you displace the block away from $x = 0$, but it is positive when the displacement vector points toward $x = 0$.

Introduction to Potential Energy

The work energy theorem: $W_{total} = K_f - K_i$

A) A force acting on a particle over a **distance** changes the **kinetic** energy of the particle.
B) To calculate the change energy, you must know the force as a function of **distance.**
C) To illustrate the work-energy concept, consider the case of a stone falling from x_i to x_f under the influence gravity.
Using the work-energy concept, we say that work is done by the gravitational **force**, resulting in an increase of the **kinetic** energy of the stone.

The work of conservative forces can be written in terms of the change in potential energy:

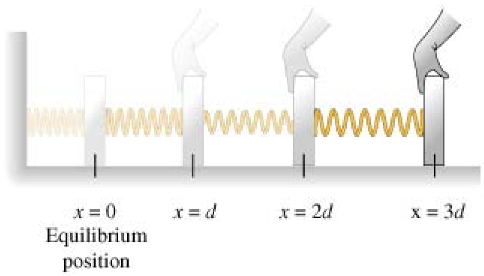
$$W_c = -\Delta U = -(U_f - U_i)$$
$$W_{total} = W_{nc} + W_c = \Delta K$$
$$W_{nc} - \Delta U = \Delta K$$
$$W_{nc} = \Delta K + \Delta U = \Delta E$$

D) Rather than ascribing the increased kinetic energy of the stone to the work of gravity, we now (when using potential energy rather than work-energy) say that the increased kinetic energy comes from the **change** of the **potential** energy.

E) This process happens in such a way that *total mechanical energy*, equal to the **sum** of the kinetic and potential energies, is **conserved.**

Stretching a Spring

As illustrated in the figure, a spring with spring constant k is stretched from (x=0) to (x=3d) , where (x=0) is the equilibrium position of the spring. (Intro 1 figure)



A)Remember that the amount of work done by the spring is equal to the negative of the change in potential energy. The work done by us is opposite to the one of the spring.
$W_{hand}=-W_{spring}=U_{f}-U_{i}$

From x=0 to x=d: $W_{hand} = \frac{1}{2}k(d)^2 - \frac{1}{2}k(0)^2 = \frac{1}{2}kd^2$

From x=d to x=2d: $W_{hand} = \frac{1}{2}k(2d)^2 - \frac{1}{2}k(d)^2 = \frac{3}{2}kd^2$

From x=2d to x=3d: $W_{hand} = \frac{1}{2}k(3d)^2 - \frac{1}{2}k(2d)^2 = \frac{5}{2}kd^2$

B) The same energy is needed to compress or stretch the spring.

From x=0 to x=d: $W_{hand} = \frac{1}{2}k(d)^2 - \frac{1}{2}k(0)^2 = \frac{1}{2}kd^2$

From x=0 to x=-d: $W_{hand} = \frac{1}{2}k(-d)^2 - \frac{1}{2}k(0)^2 = \frac{1}{2}kd^2$

C) Now consider two springs A and B that are attached to a wall. Spring A has a spring constant that is four times that of the spring constant of spring B. If the same amount of energy is required to stretch both springs, what can be said about the distance each spring is stretched?

$k_A = 4k_B$ Given condition

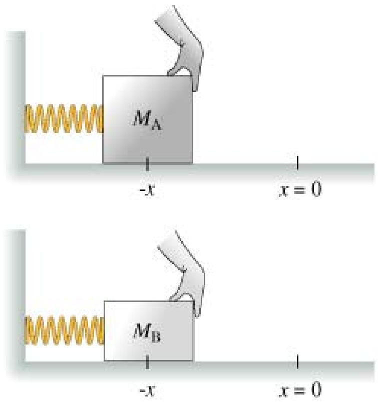
$\frac{1}{2}k_A x_A^2 = \frac{1}{2}k_B x_B^2$ The energies are equal

$4k_B x_A^2 = k_B x_B^2$

$2x_A = x_B$

Spring A must stretch half the distance spring B stretches.

D) Two identical springs are attached to two different masses, M_A and M_B, where M_A is greater than M_B. The masses lie on a frictionless surface. Both springs are compressed the same distance, d, as shown in the figure. Which of the following statements describes the energy required to compress spring A and spring B? (Part D figure)



The energy associated with a spring is independent of mass. Spring A requires the same amount of energy as spring B.

8.10 Find the gravitational potential energy of an 88 kg person standing atop Mt. Everest at an altitude of 8848 m. Use sea level as the location for y=0.

10. **Picture the Problem**: The climber stands at the top of Mt. Everest.

Strategy: Find the gravitational potential energy by using equation 8-3.

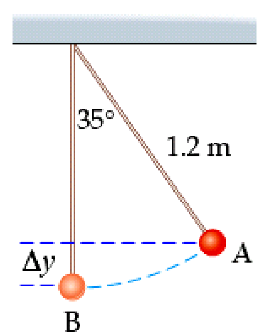
Solution: Calculate $U = mgy$: $U = mgy = (88\text{ kg})(9.81\text{ m/s}^2)(8848\text{ m}) = 7.6\times10^6\text{ J} = \boxed{7.6\text{ MJ}}$

Insight: You are free to declare that the climber's potential energy is zero at the top of Mt. Everest and −7.2 MJ at sea level!

8.17 A 0.33 kg pendulum bob is attached to a string 1.2 m long.

What is the change in the gravitational potential energy of the system as the bob swings from point A to point B in the figure

17. **Picture the Problem**: The pendulum bob swings from point A to point B and loses altitude and thus gravitational potential energy. See the figure at right.



Strategy: Use the geometry of the problem to find the change in altitude Δy of the pendulum bob, and then use equation 8-3 to find its change in gravitational potential energy.

Solution: 1. Take the apex of the Pendulum as the place for y=0. Let L be the length of the pendulum.

$$y_i = y_A = -L\cos\theta$$

$$y_f = y_B = -L$$

2. Find ΔU:

$$\Delta U = U_f - U_i$$

$$\Delta U = mgy_f - mgy_i$$

$$\Delta U = mg(-L - (-L\cos\theta))$$

$$\Delta U = mgL(\cos\theta - 1)$$
$$= (0.33\text{ kg})(9.81\text{ m/s}^2)(1.2\text{ m})(\cos 35° - 1)$$
$$\Delta U = \boxed{-0.70\text{ J}}$$

Insight: Note that the change in height Δy is negative because the pendulum swings from A to B. Likewise, the change in height is positive and the pendulum gains potential energy if it swings from B to A.

Conservation of Energy Ranking Task

A) Rank each pendulum on the basis of its initial gravitational potential energy (before being released).

Possible combinations for potential energy

a) m=8 kg; h=15 cm; $U = (8)(9.8)(0.15) = 11.76\text{ J}$
b) m=1 kg; h=60cm; $U = (1)(9.8)(0.6) = 5.88\text{ J}$
c) m=2 kg; h=60 cm; $U = (2)(9.8)(0.6) = 11.76\text{ J}$
d) m=3 kg; h=45 cm; $U = (3)(9.8)(0.45) = 13.23\text{ J}$
e) m=4 kg; h=30 cm; $U = (4)(9.8)(0.3) = 11.76\text{ J}$
f) m=2 kg; h=30 cm; $U = (2)(9.8)(0.3) = 5.88\text{ J}$

From largest to smallest: d, (a,c,d), (b,f)

B) Rank each pendulum on the basis of the maximum kinetic energy it attains after release.

Energy is conserved. All the potential energy converts into kinetic, when the maximum kinetic is reached. So the order of the pendulums according to possible kinetic energies is the same as above.

C) Rank each pendulum on the basis of its maximum speed.
Converting the potential into kinetic energy

$$\frac{1}{2}mv^2 = mgh$$
$$v^2 = 2gh$$

The larger the height the larger the speed, independent of the mass.

8.42 Starting at rest at the edge of a swimming pool, a 72.0 kg athlete swims along the surface of the water and reaches a speed of 1.20 m/s by doing the work W_{nc1} = 161 J .

Find the nonconservative work, W_{nc2} , done by the water on the athlete.

42. **Picture the Problem**: The athlete accelerates horizontally through the water from rest to 1.20 m/s while doing nonconservative work against the drag from the water.

Strategy: The total nonconservative work done on the athlete changes his mechanical energy according to equation 8-9. This nonconservative work includes the positive work W_{nc1} done by the athlete's muscles and the negative work W_{nc2} done by the water. Use this relationship and the known change in kinetic energy to find W_{nc2}.

Solution: Set the nonconservative work equal to the change in mechanical energy and solve for W_{nc2}. The initial mechanical energy is zero:

$$W_{\text{nc}} = W_{\text{nc1}} + W_{\text{nc2}} = \Delta E = E_{\text{f}} - E_{\text{i}} = K_{\text{f}} - 0$$
$$W_{\text{nc2}} = K_{\text{f}} - W_{\text{nc1}} = \tfrac{1}{2}mv_{\text{f}}^2 - W_{\text{nc1}}$$
$$= \tfrac{1}{2}(72.0\text{ kg})(1.20\text{ m/s})^2 - (161\text{ J}) = \boxed{-109\text{ J}}$$

Insight: The drag force from the water reduced the swimmer's mechanical energy, but his muscles increased it by a greater amount, resulting in a net gain in mechanical energy.

8.50 An 81.0 kg in-line skater does 3420 J of nonconservative work by pushing against the ground with his skates. In addition, friction does -715 J of nonconservative work on the skater. The skater's initial and final speeds are 2.50 m/s and 1.22 m/s, respectively.

50. **Picture the Problem**: The skater travels up a hill (we know this for reasons given below), changing his kinetic and gravitational potential energies, while both his muscles and friction do nonconservative work on him.

Strategy: The total nonconservative work done on the skater changes his mechanical energy according to equation 8-9. This nonconservative work includes the positive work W_{nc1} done by his muscles and the negative work W_{nc2} done by the friction. Use this relationship and the known change in potential energy to find Δy.

Solution: 1. (a) The skater has gone uphill because the work done by the skater is larger than that done by friction, so the skater has gained mechanical energy. However, the final speed of the skater is less than the initial speed, so he has lost kinetic energy. Therefore he must have gained potential energy, and has gone uphill.

2. (b) Set the nonconservative work equal to the change in mechanical energy and solve for Δy:

$$W_{\text{nc}} = W_{\text{nc1}} + W_{\text{nc2}} = \Delta E = E_{\text{f}} - E_{\text{i}}$$
$$W_{\text{nc1}} + W_{\text{nc2}} = (K_{\text{f}} + U_{\text{f}}) - (K_{\text{i}} + U_{\text{i}}) = \tfrac{1}{2}m(v_{\text{f}}^2 - v_{\text{i}}^2) + mg\Delta y$$
$$\Delta y = \left[W_{\text{nc1}} + W_{\text{nc2}} - \tfrac{1}{2}m(v_{\text{f}}^2 - v_{\text{i}}^2)\right]\Big/ mg$$
$$= \frac{\left\{(3420\text{ J}) + (-715\text{ J}) - \tfrac{1}{2}(81.0\text{ kg})\left[(1.22\text{ m/s})^2 - (2.50\text{ m/s})^2\right]\right\}}{(81.0\text{ kg})(9.81\text{ m/s}^2)} = \boxed{3.65\text{ m}}$$

Insight: Verify for yourself that if the skates had been frictionless but the skater's muscles did the same amount of work, the skater's final speed would have been 4.37 m/s. He would have sped up if it weren't for friction!

8.30 A 2.9 kg block slides with a speed of 1.6 m/s on a frictionless horizontal surface until it encounters a spring.

A) If the block compresses the spring 4.8 cm before coming to rest, what is the force constant of the spring?

B) What initial speed should the block have to compress the spring by 1.2cm?

30. **Picture the Problem**: A block slides on a frictionless, horizontal surface and encounters a horizontal spring. It compresses the spring and briefly comes to rest.

Strategy: Set the mechanical energy when sliding freely equal to the mechanical energy when the spring is fully compressed and the block is at rest. Solve the resulting equation for the spring constant *k*, then repeat the procedure to find the initial speed required to compress the spring only 1.2 cm before coming to rest.

Solution: 1. (a) Set $E_i = E_f$ where the initial state is when it is sliding freely and the final state is when it is at rest, having compressed the spring.

$$K_i + U_i = K_f + U_f$$
$$\tfrac{1}{2}mv_i^2 + 0 = 0 + \tfrac{1}{2}kx_{max}^2$$
$$k = \frac{mv_i^2}{x_{max}^2} = \frac{(2.9\text{ kg})(1.6\text{ m/s})^2}{(0.048\text{ m})^2} = 3200\text{ N/m} = \boxed{3.2\text{ kN/m}}$$

2. (b) Solve the equation from step 1 for v_i:

$$v_i = \sqrt{\frac{kx_{max}^2}{m}} = \sqrt{\frac{(3200\text{ N/m})(0.012\text{ m})^2}{2.9\text{ kg}}} = \boxed{0.40\text{ m/s}}$$

Insight: The kinetic energy of the sliding block is stored as potential energy in the spring. Moments later the spring will have released all its potential energy, the block would have gained its kinetic energy again, and would then be sliding at the same speed but in the opposite direction.

8.49 A 1250 kg car drives up a hill that is 16.2 m high. During the drive, two nonconservative forces do work on the car: (i) the force of friction, and (ii) the force generated by the car's engine. The work done by friction is -3.11×10^5 J ; the work done by the engine is 6.44×10^5 J .

49. **Picture the Problem**: The car drives up the hill, changing its kinetic and gravitational potential energies, while both the engine force and friction do nonconservative work on the car.

Strategy: The total nonconservative work done on the car changes its mechanical energy according to equation 8-9. This nonconservative work includes the positive work W_{nc1} done by the engine and the negative work W_{nc2} done by the friction. Use this relationship and the known change in potential energy to find ΔK.

Solution: Set the nonconservative work equal to the change in mechanical energy and solve for ΔK:

$$W_{nc} = W_{nc1} + W_{nc2} = \Delta E = E_f - E_i$$
$$W_{nc1} + W_{nc2} = (K_f + U_f) - (K_i + U_i) = \Delta K + mg(y_f - y_i)$$
$$\Delta K = W_{nc1} + W_{nc2} - mg(y_f - y_i)$$
$$= (6.44\times10^5\text{ J}) + (-3.11\times10^5\text{ J}) - (1250\text{ kg})(9.81\text{ m/s}^2)(16.2\text{ m})$$
$$\Delta K = 1.34\times10^5\text{ J} = \boxed{134\text{ kJ}}$$

Insight: The friction force reduces the car's mechanical energy, but the engine increased it by a greater amount, resulting in a net gain in both kinetic and potential energy. The car gained speed while traveling uphill.