Physics 100A Homework 5 – Chapter 6

Contact Forces Introduced

A) When two objects slide against one another, the magnitude of the frictional force is always equal to $\mu_k N$

B) When two objects are in contact with no relative motion, the magnitude of the frictional force may be either less or equal to $\mu_s N$

C) When a board with a box on it is slowly tilted to a larger and larger angle, common experience shows that the box will at some point "break loose" and start to accelerate down the board. The box begins to slide once the component of its weight parallel to the board, $W_\parallel$, equals the maximum force of static friction. The box *accelerates* down the board after it begins to slide because the coefficient of kinetic friction is less than the coefficient of static friction.

Comparing Friction Forces with Gravity

A) The crate is at rest on the incline because the frictional force points up the incline.

B) When the physicist attempts to move the create by pushing up the incline, he finds that the force of static friction points down the incline. Constant velocity

C) When the two physicists push the crate up the incline with constant velocity, their combined force is the same as the sum of the force of gravity parallel to the incline plus the force of kinetic friction. Both forces pointing down the incline.

Kinetic Friction in a Block-and-Pullet System.
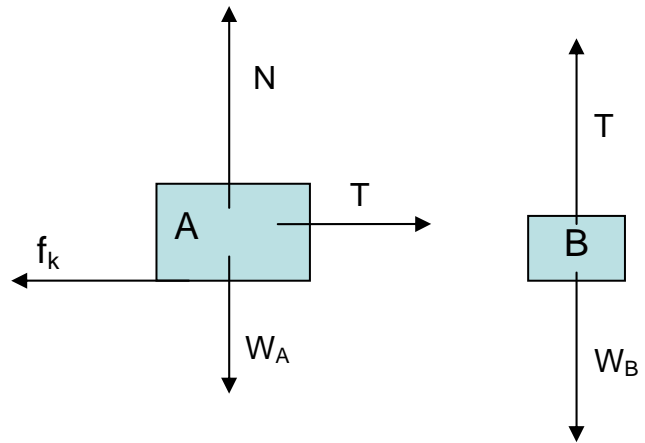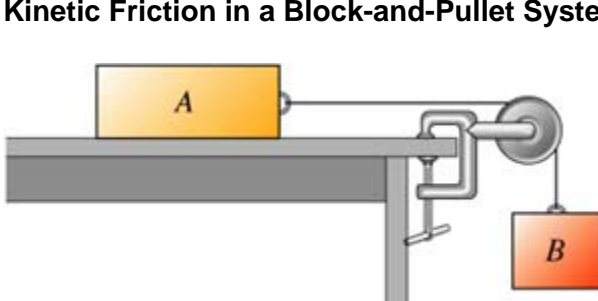
Consider the system shown in the figure. Block A has weight 4.91 N and block B has weight 2.94 N . Once block B is set into downward motion, it descends at a constant speed. Assume that the mass and friction of the pulley are negligible.

Block A (no acceleration)

$$T - f_k = 0$$
$$N - W_A = 0 \qquad ----> T = \mu_k W_A$$
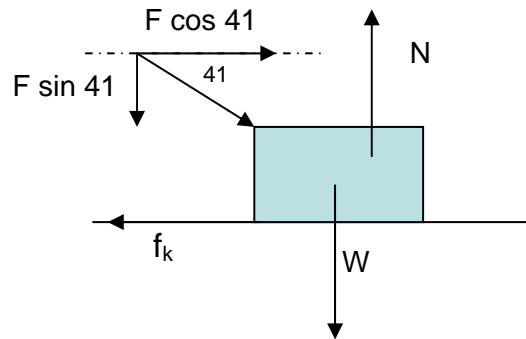$$f_k = \mu_k N = \mu_k W_A$$

Block B (no acceleration)

$$T = M_B g$$

Combining both expressions for T

$$\mu_k W_A = W_B \qquad \mu_k = \frac{W_B}{W_A} = \frac{2.94}{4.91} = 0.6$$

Pushing a Chair Along the Floor

A chair of weight 80.0 N lies atop a horizontal floor; the floor is not frictionless. You push on the chair with a force of F= 42.0 N directed at an angle of 41.0° below the horizontal and the chair slides along the floor.
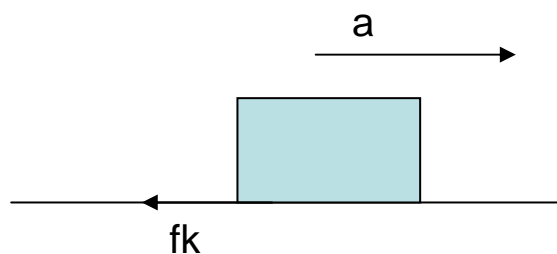

Vertical direction

$$N - F\sin(41) - W = 0 \qquad N = W + F\sin(41) = 80 + 42\sin(41) = 108 \text{ N}$$

6.3 A baseball player slides into third base with an initial speed of 4.0 m/s.

If the coefficient of kinetic friction between the player and the ground is 0.46, how far does the player slide before coming to rest?

Horizontal direction. The only force is that of friction. $\qquad -f_k = ma \qquad a = -\frac{f_k}{m} = -\frac{\mu_k mg}{m} = -\mu_k g$

3. **Picture the Problem**: A baseball player slides in a straight line and comes to rest due to the frictional force.

    Strategy: Find the acceleration of the player using Newton's Second Law, and insert the result into equation 2-12 to find the slide distance. Let the initial velocity $\vec{v}_0$ point in the positive direction.

    Solution: 1. Find the acceleration of the
    player using Newton's Second Law:

    2. Use equation 2-12 to find the slide distance: $\Delta x = \dfrac{v^2 - v_0^2}{2a} = \dfrac{0 - v_0^2}{-2\mu_k\, g} = \dfrac{(4.0 \text{ m/s})^2}{2(0.46)(9.81 \text{ m/s}^2)} = \boxed{1.8 \text{ m}}$

    Insight: If the player were running faster, say, 8.0 m/s, a μ_k of 0.46 would result in a slide distance of 7.1 m, a quarter of the 27 m between the bases! He might very well overshoot the base and be tagged out.

6.12 A 45-kg crate is placed on an inclined ramp. When the angle the ramp makes with the horizontal is increased to 26°, the crate begins to slide downward.

12. **Picture the Problem**: The crate slides down the incline when the angle is **just** 26°. A free-body diagram of the situation is depicted at right.

    Strategy: Choose the *x*-axis to be parallel to the ramp, pointing uphill, and the *y* axis to point perpendicular to the ramp. Write Newton's Second Law in the *y* direction to find the normal force *N*. Then write Newton's Second Law in the *x* direction with $a_x = 0$ (the box just begins to slide) to find the coefficient of static friction μ_s.

    Solution: 1. (a) Write Newton's
    Second Law in the *y* direction: $\sum F_y = N - mg\cos\theta = 0$
                                          $N = mg\cos\theta$

    2. Write Newton's Second Law in the *x* $\sum F_x = \mu_s N - mg\sin\theta = ma_x = 0$
    direction, setting $a_x = 0$ (the box just $\mu_s(mg\cos\theta) - mg\sin\theta = 0$
    begins to slide) and solving for μ_s:
                                                      $\mu_s = \tan\theta = \tan 26° = \boxed{0.49}$

    3. (b) Since θ only depends upon μ_s, changing the mass will not change θ and the angle remains $\boxed{26°}$.
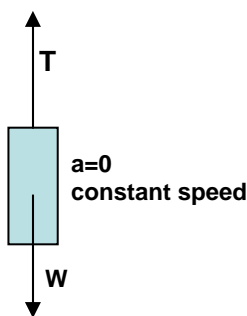
    Insight: Increasing the mass does increase the normal force and hence the friction force, but the component of the crate's weight that pulls it down the ramp is also increased. The two effects cancel and θ depends upon μ_s only.
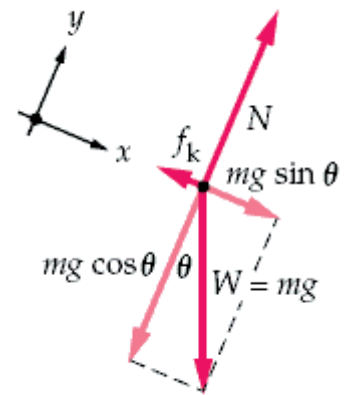
Lifting a Bucket

A 6-kg bucket of water is being pulled straight up by a string at a constant speed.

A) The tension in the rope

$T - W = 0 \qquad T = mg = (6)(9.8) = 59 \text{ N, about 60N}$

B) At a certain point the speed of the bucket begins to change. The bucket now has an upward constant acceleration of magnitude 3m/s^2. What is the tension in the rope now?

$$T - W = ma \qquad T = mg + ma = m(g + a) = (6)(9.8 + 3) = 76.8 \text{ N, about 76N}$$

C) If the acceleration is downward.

$$T - W = -ma \qquad T = mg - ma = m(g - a) = (6)(9.8 - 3) = 40.8 \text{ N, about 42N}$$

6.31) A picture hangs on the wall suspended by two strings, as shown in the figure . The tension in string 1 is 1.7 N.

31. **Picture the Problem**: The free-body diagram for the contact point between the two strings is depicted at right.

Strategy: The horizontal components of the string tensions must be equal because the picture is not accelerating. The same is true of the vertical components of the forces. Use Newton's Second Law in the horizontal direction to find the tension in string 2, and in the vertical direction to find the weight of the picture.

Solution: 1. (a) The tension in string 2 is less than the tension in string 1, because it provides mostly a sideways component of force that is balanced by the horizontal component of string 1. That means string 1 must support most of the weight of the picture plus balance string 2's horizontal component, giving it a larger tension than string 2.

2. (b) Write Newton's Second Law in the horizontal direction in order to find the tension in string 2:

$$\sum F_x = -T_1 \cos \theta_1 + T_2 \cos \theta_2 = 0$$

$$T_2 = T_1 \frac{\cos \theta_1}{\cos \theta_2} = (1.7 \text{ N}) \frac{\cos 65°}{\cos 32°} = \boxed{0.85 \text{ N}}$$

3. (c) Write Newton's Second Law in the vertical direction in order to find the picture's weight:

$$\sum F_y = T_1 \sin \theta_1 + T_2 \sin \theta_2 - W = 0$$

$$W = (1.7 \text{ N}) \sin 65° + (0.85 \text{ N}) \sin 32° = \boxed{2.0 \text{ N}}$$

Insight: As the angle of string 1 approaches 90° and the angle of string 2 approaches 0°, the tension in string 2 drops to zero and the entire 2.0 N weight of the picture is supported by string 1.

6.37) Two blocks are connected by a string, as shown in the figure. The smooth inclined surface makes an angle of 42 °with the horizontal, and the block on the incline has a mass of 6.7kg.

37. **Picture the Problem**: The forces exerted on the left and right blocks are depicted at right.


Strategy: Because the pulley is ideal, the tension in the string is equal to the weight of the hanging block. This can be verified by Newton's Second Law in the vertical direction for the hanging block: $\sum F_y = T - mg = 0$. Write Newton's Second Law along the direction parallel to the incline for the 6.7 kg block and substitute $T = mg$ into the resulting equation to find m.

Solution: 1. Write Newton's Second Law along the direction parallel to the incline:

$$\sum F_x = T - Mg \sin \theta = 0$$

2. Substitute $T = mg$ into the resulting equation to find m.

$$T = mg = Mg \sin \theta$$

$$m = M \sin \theta = (6.7 \text{ kg}) \sin 42° = \boxed{4.5 \text{ kg}}$$
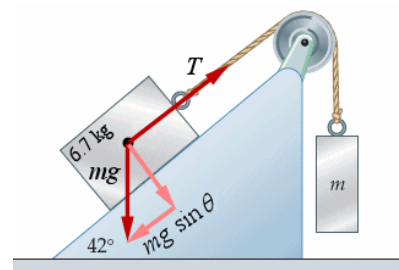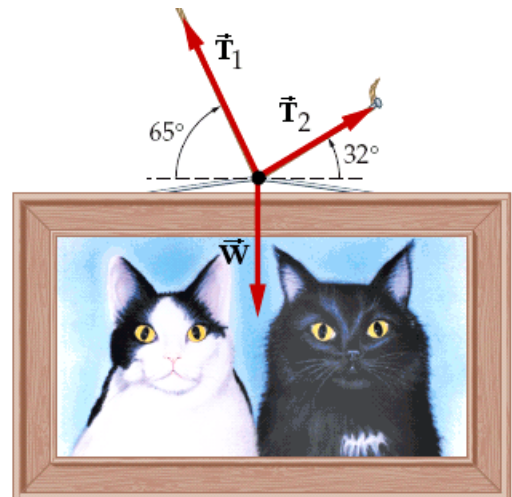
Insight: A larger m is required if the angle θ is increased. If it is increased all the way to $\theta = 90°$, the large mass will be hanging straight down and the mass m required to maintain equilibrium would be 6.7 kg.

2. (b) The tension in the string was determined in Example 6-6 to be $T = m_1 m_2 g/(m_1 + m_2)$. If both masses were doubled the tension would be $T_{\text{new}} = 2m_1 2m_2 g/(2m_1 + 2m_2) = 2T$. We conclude that if both masses were doubled the tension would increase.

Insight: The tension must increase in the second case because there is twice as much mass that must be accelerated at the same rate as before.

6.44) Find the acceleration of the masses shown in the figure, given that $m_1 = 1.0kg, m_2 = 2.0kg$, and $m_3 = 3.0kg$. Assume the table is frictionless and the masses move freely.

44.   **Picture the Problem**: The physical apparatus is shown at right.

Strategy: Write Newton's Second Law for each of the three blocks and add the equations to eliminate the unknowns T_1 and T_2. Solve the resulting equation for the acceleration a. Let x be positive in the direction of each mass's motion.
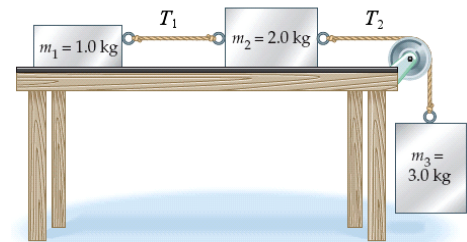
Solution: 1. Write Newton's Second Law for each of the three blocks and add the equations:

$$\sum_{\text{block 1}} F_x = \ T_1 \qquad\qquad = m_1 a$$

$$\sum_{\text{block 2}} F_x = -T_1 + T_2 \qquad = m_2 a$$

$$\sum_{\text{block 3}} F_x = \qquad -T_2 + m_3 g = m_3 a$$

$$m_3 g = (m_1 + m_2 + m_3)a$$

2. Solve the resulting equation for a:

$$a = \left(\frac{m_3}{m_1 + m_2 + m_3}\right)g = \frac{3.0\text{ kg}}{6.0\text{ kg}}(9.81\text{ m/s}^2) = \boxed{4.9\text{ m/s}^2}$$

Insight: Note that the blocks move as if they were a single block of mass 6.0 kg under the influence of a force equal to $m_3 g = 29$ N.

Insight: A comet that travels along an elliptical path experiences the largest gravitational force when it is closest to the Sun. In such a case the required centripetal force is largest both because v is greatest and r is least at that point. In orbit terminology the point closest to the Sun is called perihelion.

A Mass in a Turntable
A small metal cylinder rests on a circular turntable that is rotating at a constant rate, as illustrated in the diagram.
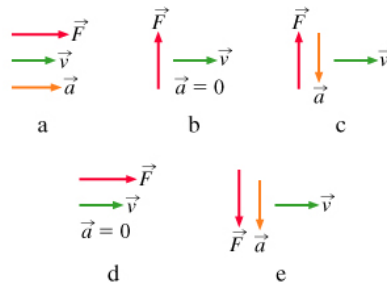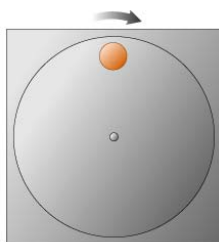
A) In circular motion with constant speed the acceleration and force point radially inward and the velocity is tangential. (e)

B) The tangential speed is $v = r\omega$, and the centripetal acceleration is $a_{cp} = r\omega^2$, where the angular velocity ω is constant. If the radius is halved, the speed and the acceleration are halved.

6.56) Find the linear speed of the bottom of a test tube in a centrifuge if the centripetal acceleration there is 5.2×10^4 times the acceleration of gravity. The distance from the axis of rotation to the bottom of the test tube is 7.5 cm.

56. **Picture the Problem**: The test tube travels along a circular path at constant speed.

   Strategy: Solve equation 6-15 for the speed required to attain the desired acceleration.

   Solution: Solve equation 6-15 for the speed: $v = \sqrt{ra_{cp}} = \sqrt{r(52,000g)} = \sqrt{(0.075 \text{ m})(52,000)(9.81 \text{ m/s}^2)}$
   $$= 200 \text{ m/s} = \boxed{0.20 \text{ km/s}}$$

   Insight: This speed corresponds to 25,000 revolutions per minute for the centrifuge, or 415 revolutions per second.

   Insight: The tension does not quadruple in part (a) because the acceleration is the sum of gravitational and centripetal acceleration. The centripetal acceleration quadruples, but the sum does not because gravity remains constant.

To calculate the number of revolutions above:

$$\omega = \frac{v}{r} = \frac{200}{0.075} = 2,666 \text{ rad/s} \qquad \omega = 2\pi f \qquad f = \frac{\omega}{2\pi} = \frac{2,666}{2\pi} = 424 \text{ rev/s}$$

6.62 Driving in your car with a constant speed of 12m/s, you encounter a bump in the road that has a circular cross-section, as indicated in the figure .
If the radius of curvature of the bump is 35m, find the apparent weight of a 67-kg person in your car as you pass over the top of the bump.

62. **Picture the Problem**: The car follows a circular path at constant speed as it passes over the bump.

   Strategy: The centripetal acceleration is downward, toward the center of the circle, as the car passes over the bump. Write Newton's Second Law in the vertical direction and solve for the normal force N, which is also the apparent weight of the passenger.

   Solution: 1. Write Newton's Second Law for the passenger and solve for N:
   $$\sum F_y = N - mg = -ma_{cp} = -mv^2/r$$
   $$N = m(g - v^2/r)$$

   2. Insert numerical values: $N = (67 \text{ kg})\left[9.81 \text{ m/s}^2 - \frac{(12 \text{ m/s})^2}{35 \text{ m}}\right] = 380 \text{ N} = \boxed{0.38 \text{ kN}}$

   Insight: This apparent weight is 42% less than the normal 0.66-kN weight of the passenger.