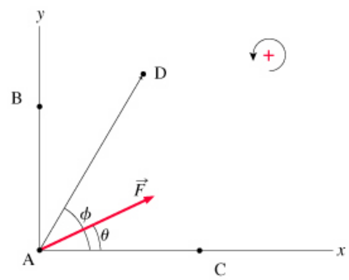
Physics 100A Homework 11- Chapter 11 (part 1)

Finding Torque

A force $\vec{F}$ of magnitude F making an angle θ with the *x* axis is applied to a particle located along axis of rotation A, at Cartesian coordinates $(0,0)$ in the figure. The vector $\vec{F}$ lies in the *xy* plane, and the four axes of rotation A, B, C, and D all lie perpendicular to the *xy* plane.



A) What is the torque τ_A about axis A due to the force $\vec{F}$?

The force passes through the point A, so there is no arm and the torque is zero.

B) What is the torque τ_B about axis B due to the force $\vec{F}$? (B is the point at Cartesian coordinates $(0,b)$, located a distance b from the origin along the *y* axis.)

The magnitude of the torque is $\tau = rF_\perp$ where $F_\perp$ is the component of the force perpendicular to r .

Note that the torque can also be calculated with the concept of the moment arm $\tau = r_\perp F$, where $r_\perp$ is the component of the radial vector perpendicular to the direction of the force.
The answer is the same. In most problems it is easier to visualize the perpendicular component of the force. We will use what is simplest in a given problem.

$r = b \qquad F_\perp = F\cos\theta$

The direction of the torque follows the convention: positive if it produces a counterclockwise rotation and negative if it produces a clockwise rotation.

$\tau_B = bF\cos\theta$

C) What is the torque τ_C about axis C due to the force $\vec{F}$? (C is the point at Cartesian coordinates $(0,c)$, located a distance c from the origin along the *x* axis.)

$r = c \qquad F_\perp = F\sin\theta$ The torque produces a clockwise rotation about C.

$\tau_c = -cF\sin\theta$

D) What is the torque τ_D about axis D due to the force $\vec{F}$? (D is the point located a distance d from the origin and making an angle ϕ with the x axis.)

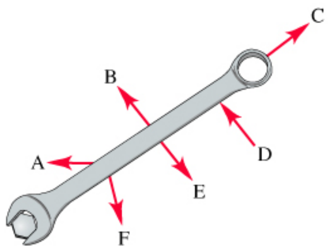
The angle between $\vec{F}$ and the $\vec{r}$ vector AD is ($\phi - \theta$).

$F_\perp = F\sin(\phi - \theta)$ and the rotation is counterclockwise

$\tau_D = dF\sin(\phi - \theta)$

Torque Magnitude Ranking Task

The wrench in the figure has six forces of equal magnitude acting on it.

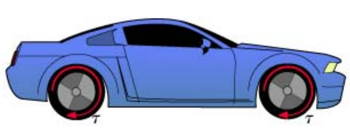


Rank these forces (A through F) on the basis of the magnitude of the torque they apply to the wrench, measured about an axis centered on the bolt.

The larger the radius, the larger the torque magnitude. The larger the perpendicular projection onto the radial direction, the larger the torque. Then in decreasing order: D, (B,E), F, A, C

Torque!

The car shown in the figure has mass m (this includes the mass of the wheels). The wheels have radius r, mass m_W, and moment of inertia $I = km_W r^2$. Assume that the axles apply the same torque τ to all four wheels. For simplicity, also assume that the weight is distributed uniformly so that all the wheels experience the same normal reaction from the ground, and so the same frictional force.



A) If there is no slipping, a frictional force must exist between the wheels and the ground. In what direction does the frictional force act? Take the positive *x* direction to be to the right.

The frictional force acts in the positive x direction.

B) Use Newton's laws to find an expression for the net external force acting on the car. Ignore air resistance.

The vertical forces balance. The net force is in the horizontal direction: $F_{total} = 4f$ where f is the friction force in each wheel.

C) Use Newton's laws to find an expression for N, the normal force on each wheel.

Forces in the y direction: $4N_{wheel} - mg = 0$ where N_{wheel} is the normal force on each wheel that we take to be equal since the weight is equally distributed.

$N_{wheel} = mg / 4$

D) Now assume that the frictional force f is not at its maximum value. What is the relation between the torque τ applied to each wheel by the axles and the acceleration a of the car? Once you have the exact expression for the acceleration, make the approximation that the wheels are much lighter than the car as a whole.

The torque force at each wheel is: $\tau = rf$.

The linear acceleration is obtained from the sum of the forces $4f = ma$

$f = \tau / r$ and $f = ma/4$, then ($ma/4 = \tau / r$) and $a = \dfrac{4\tau}{mr}$

A person slowly lowers a 3.6 kg crab trap over the side of a dock, as shown in the figure.

What torque does the trap exert about the person's shoulder?

Picture the Problem: The arm extends out either horizontally and the weight of the crab trap is exerted straight downward on the hand.

Strategy:. In this case the downward gravitational force is perpendicular to the radial direction. The rotation in counterclockwise.

Solution: The torque: $\tau = rmg = (0.70\text{ m})(3.6\text{ kg})(9.81\text{ m/s}^2) = \boxed{25\text{ N}\cdot\text{m}}$

Insight: If the man bent his elbow and brought his hand up next to his shoulder, the torque on the shoulder would be zero but the force on his hand would remain 35 N or 7.9 lb.

11.11) When a ceiling fan rotating with an angular speed of 2.90 rad/s is turned off, a frictional torque of 0.110 N·m slows it to a stop in 19.0 s.

Picture the Problem: The ceiling fan rotates about its axis, decreasing its angular speed at a constant rate.

Strategy: Determine the angular acceleration using equation 10-6 and then use equation 11-4 to find the moment of inertia of the fan.

Solution: Solve equation 11-4 for *I*: $I = \dfrac{\tau}{\alpha} = \dfrac{\tau}{\Delta\omega/\Delta t} = \dfrac{\tau\,\Delta t}{\Delta\omega} = \dfrac{(-0.120\text{ N}\cdot\text{m})(22.5\text{ s})}{(0-2.75\text{ rad/s})} = \boxed{0.982\text{ kg}\cdot\text{m}^2}$

Insight: Friction converts the fan's initial kinetic energy of $\frac{1}{2}I\omega_0^2 = 3.10$ J into heat. Rotational work will be examined in more detail in section 11-8.

11.19) A fish takes the bait and pulls on the line with a force of 2.2 N. The fishing reel, which rotates without friction, is a uniform cylinder of radius 0.055 m and mass 0.99 kg.

A) What is the angular acceleration of the fishing reel?

B) How much line does the fish pull from the reel in 0.25 s?

Picture the Problem: The fish exerts a torque on the fishing reel and it rotates with constant angular acceleration.

Strategy: Use Table 10-1 to determine the moment of inertia of the fishing reel assuming it is a uniform cylinder ($\frac{1}{2}MR^2$). Find the torque the fish exerts on the reel by using equation 11-1. Then apply Newton's Second Law for rotation (equation 11-4) to find the angular acceleration and equations 10-2 and 10-10 to find the amount of line pulled from the reel.

Solution: 1. (a) Use Table 10-1 to find I:

$$I = \tfrac{1}{2}MR^2 = \tfrac{1}{2}(0.99\text{ kg})(0.055\text{ m})^2 = \underline{\underline{0.0015\text{ kg}\cdot\text{m}^2}}$$

2. Apply equation 11-1 directly to find τ:

$$\tau = r\,F = (0.055\text{ m})(2.2\text{ N}) = \underline{\underline{0.121\text{ N}\cdot\text{m}}}$$

3. Solve equation 11-14 for α:

$$\alpha = \frac{\tau}{I} = \frac{0.121\text{ N}\cdot\text{m}}{0.0015\text{ kg}\cdot\text{m}^2} = \boxed{81\text{ rad/s}^2}$$

4. (b) Apply equations 10-2 and 10-10:

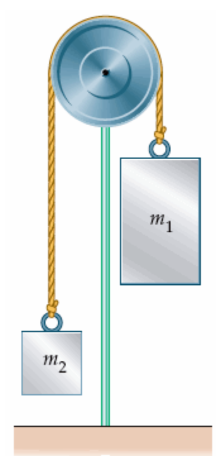
$$s = r\theta = r\left(\tfrac{1}{2}\alpha t^2\right) = (0.055\text{ m})\tfrac{1}{2}\left(81\text{ rad/s}^2\right)(0.25\text{ s})^2$$
$$= \boxed{0.14\text{ m}}$$

Insight: This must be a small fish because it is not pulling very hard; 2.2 N is about 0.49 lb or 7.9 ounces of force. Or maybe the fish is tired?

11.22) A string that passes over a pulley has a 0.321 kg mass attached to one end and a 0.635 kg mass attached to the other end. The pulley, which is a disk of radius 9.50 cm, has friction in its axle.

What is the magnitude of the frictional torque that must be exerted by the axle if the system is to be in static equilibrium?

22. **Picture the Problem**: The two masses hang on either side of a pulley.



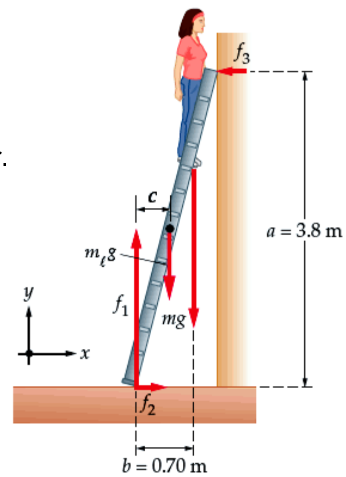
Strategy: Use Newton's Second Law for rotation (equation 11-4) to find the frictional torque τ_{fr} that would make the angular acceleration of the system equal to zero. In each case the torque exerted on the pulley by the hanging masses is the weight of the mass times the radius of the pulley. Let m_1 = 0.635 kg and m_2 = 0.321 kg. The torque due to m_1 is clockwise and therefore taken to be in the negative direction.

Solution: Write Newton's Second Law for rotation and solve for τ_{fr}:

$$\sum \vec{\boldsymbol{\tau}} = -r(m_1 g) + r(m_2 g) + \tau_{\text{fr}} = 0$$
$$\tau_{\text{fr}} = r\,g\,(m_1 - m_2)$$
$$= (0.0940\text{ m})(9.81\text{ m/s}^2)(0.635 - 0.321\text{ kg})$$
$$\tau_{\text{fr}} = \boxed{0.290\text{ N}\cdot\text{m}}$$

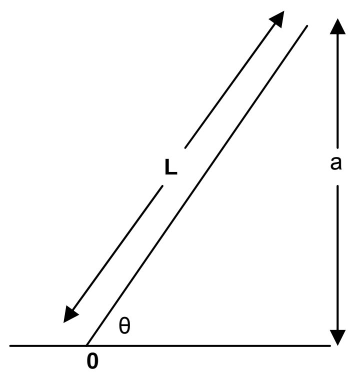
Insight: This frictional torque represents a static friction force. If a little bit of mass were added to m_1, the system would begin accelerating clockwise and the frictional torque would be reduced to its kinetic value.

11.35) An 85 kg person stands on a uniform 5.2 kg ladder that is 4.0 m long, as shown in the figure . The floor is rough; hence, it exerts both a normal force, f_1 , and a frictional force, f_2 , on the ladder. The wall, on the other hand, is frictionless; it exerts only a normal force, f_3.
Using the dimensions given in the figure, find the magnitudes of f_1 , f_2 , and f_3 .



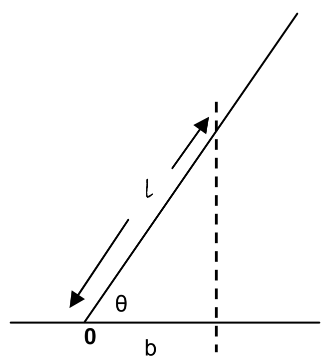
To solve the problem using the technique emphasized in class
In which we look for the perpendicular component of the force
we can calculate the angle that the ladder makes with the floor.

$$a = L\sin\theta$$
$$\theta = \sin^{-1}(a/L)$$
$$\theta = \sin^{-1}(3.8/4)$$
$$\theta = 71.8^\circ\ (71.805)$$

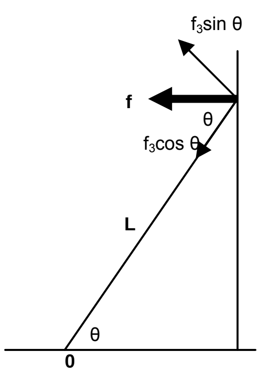


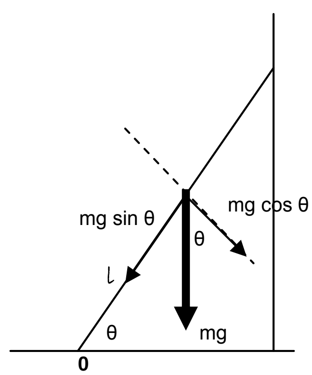
To find the length l that the person is up the ladder

$$b = l\cos\theta$$
$$l = b/\cos\theta = 0.7/\cos(71.8)$$
$$l = 2.24\ \text{m}\ \ (2.24179)$$



The torque about point 0:





$\tau_{f_3} = Lf_3\sin\theta$

For the weight of the person: $\tau_{m_p g} = -lm_p g\cos\theta$

For the weight of the ladder $\tau_{m_l g} = -(L/2)m_l g\cos\theta$

$$\sum\tau = \tau_{f_3} + \tau_{m_p g} + \tau_{m_l g} = 0$$

$$Lf_3 \sin\theta - lm_p g\cos\theta - (L/2)m_l g\cos\theta = 0$$

And

$$f_3 = \frac{(lm_p + (L/2)m_l)g\cos\theta}{L\sin\theta}$$

$$f_3 = \frac{((2.24)(85) + (2)(5.2))(9.8)\cos(71.8)}{(4)\sin(71.8)}$$

$$f_3 = \frac{((2.24)(85) + (2)(5.2))(9.81)\cos(71.8)}{(4)\sin(71.8)}$$

$$f_3 = 162 \text{ N}$$

Forces in the x-direction

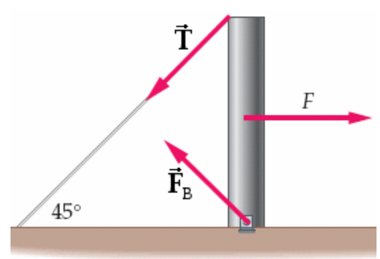
$$\sum F_x = f_2 - f_3 = 0 \qquad f_2 = f_3 = 162 \text{ N}$$

Forces in the y-direction

$$\sum F_y = f_1 - m_p g - m_l g = 0$$
$$f_1 = (m_p + m_l)g = (85 + 5.2)(9.81) = 885 \text{ N}$$

11.36) A rigid, vertical rod of negligible mass is connected to the floor by a bolt through its lower end, as shown in the figure . The rod also has a wire connected between its top end and the floor.

36. **Picture the Problem**: The horizontal force F is applied to the rod as shown in the figure at right.



Strategy: Let L = the rod length and write Newton's Second Law for torques (let the bolt be the pivot point) in order to determine the wire tension T. The pivot point is convenient since the force $\vec{\mathbf{F}}_b$ going through it produces no torque. Then write Newton's Second Law in the horizontal and vertical directions to determine the components of the bolt force $\vec{\mathbf{F}}_b$.

Solution: 1. (a) Set $\sum \tau = 0$ and solve for T:

$$\sum \tau = L\left(T\cos 45°\right) - \left(\tfrac{1}{2}L\right)F = 0$$
$$T = \frac{F}{2\cos 45°} = \boxed{\frac{F}{\sqrt{2}}}$$

2. (b) Set $\sum F_x = 0$ and solve for $F_{b,x}$:

$$\sum F_x = F + F_{b,x} - T\cos 45° = 0$$
$$F_{b,x} = T\cos 45° - F = \left(\frac{F}{2\cos 45°}\right)\cos 45° - F = \boxed{-\tfrac{1}{2}F}$$

3. (c) Set $\sum F_y = 0$ and solve for $F_{b,y}$:

$$\sum F_y = F_{b,y} - T\sin 45° = 0$$
$$F_{b,y} = \left(\frac{F}{2\cos 45°}\right)\sin 45° = \boxed{\tfrac{1}{2}F}$$

Insight: The bolt force has a magnitude of $F/\sqrt{2}$ and points 45° above the horizontal and to the left.