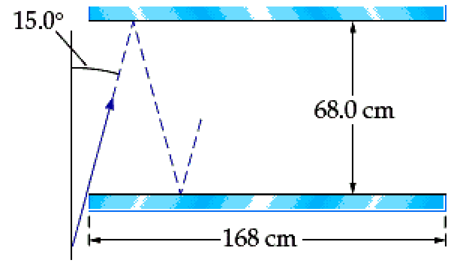
Chapter 26: Geometrical Optics

8. The image shows a light beam that is oriented 15.0° from the vertical as it reflects back and forth between two mirrors.



Let d equal the horizontal distance traveled by the light between reflections off either mirror. Calculate the distance d by multiplying the separation distance by the tangent of the beam angle. Divide the total distance (168 cm) by d to calculate the total number of reflections. From this result calculate the number of reflections off each mirror, where the first reflection is off the top mirror and the reflections alternate between the mirrors.

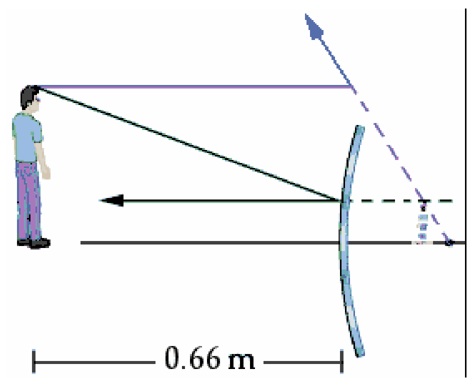
1. (a) Calculate the distance d: $d = (68.0 \text{ cm})\tan(15°) = 18.2 \text{ cm}$

2. Divide the horizontal distance by d: $N = \dfrac{x}{d} = \dfrac{168 \text{ cm}}{18.2 \text{ cm}} = 9.23 \Rightarrow$ 9 reflections

3. Because all of the odd-numbered reflections are off the top mirror (1, 3, 5, 7, and 9) the light will reflect 5 times off of the top mirror.

4. (b) All of the even-numbered reflections are off the bottom mirror (2, 4, 6, and 8), so the light will reflect 4 times off of the bottom mirror.

34. The image shows a 1.7-m-tall person standing 0.60 m from a reflecting globe with diameter 0.16 m.



Use equation 26-2 to calculate the focal length of the globe, where the radius is one half of the diameter. Then use equation 26-6 to calculate the image distance from the focal length and object distance. Finally, use equation 26-4 to calculate the height of the image.

1. (a) Calculate the focal length:

$$f = -\tfrac{1}{2}R = -\tfrac{1}{2}(D/2)$$
$$= -\tfrac{1}{2}0.18 \text{ m}/2$$
$$f = -0.045 \text{ m}$$

2. Use equation 26-4 to calculate the image distance:

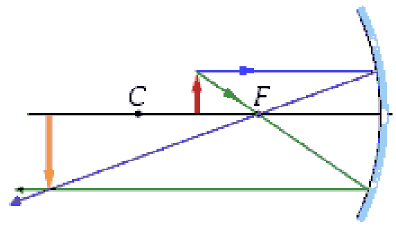
$$d_i = \left(\frac{1}{f} - \frac{1}{d_o}\right)^{-1} = \left(\frac{1}{-0.045 \text{ m}} - \frac{1}{0.66 \text{ m}}\right)^{-1} = -4.2 \text{ cm}$$

The image is 4.2 cm behind the surface of the globe.

3. (b) Calculate the image height:

$$h_i = -\frac{d_i}{d_o}h_o = -\frac{-0.042 \text{ m}}{0.66 \text{ m}}(1.7 \text{ m}) = \boxed{11 \text{ cm}}$$

38. A concave mirror produces a real, inverted, and magnified image of an object.



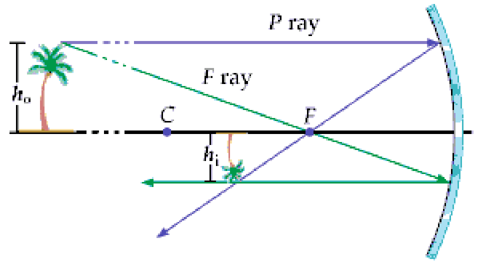
Use the magnification equation (equation 26-8) to calculate the image distance from the mirror. Then solve equation 26-6 for the focal length.

1. (a) Calculate the image distance: $d_i = -md_o = -(-3)(22 \text{ cm}) = \boxed{66 \text{ cm}}$

2. (b) Calculate the focal length from equation 26-6:

$$f = \left(\frac{1}{d_o} + \frac{1}{d_i} \right)^{-1} = \left(\frac{1}{22\text{ cm}} + \frac{1}{66\text{ cm}} \right)^{-1} = \boxed{17\text{ cm}}$$

41. The figure shows a tree located 23 m from a mirror and its real, inverted 3.8-cm-tall image located 7.0 cm in front of the mirror.

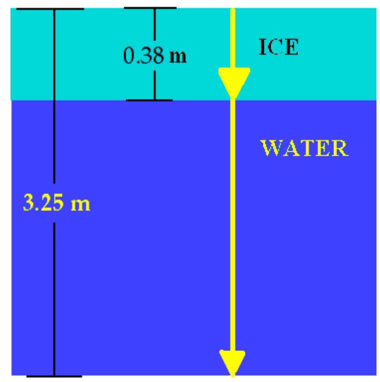


Use equation 26-4 to calculate the object height from the image height and the object and image distances.

Solve equation 26-4 for the object height:

$$h_o = -\frac{d_o h_i}{d_i} = -\frac{23\text{ m}(-3.8\text{ cm})}{7.0\text{ cm}} = \boxed{12\text{ m}}$$

57. The image shows a pond of total thickness 3.25 meters that has a 0.38 m layer of ice on the surface. Light travels from the top of the pond to the bottom.



Calculate the depth of the water by subtracting the depth of the ice from the total depth. Calculate the time for light to travel through the ice (and then through the water) by dividing the distance by the speed of light in that medium. Use equation 26-10 to calculate the speed of light in each medium, using the indices of refraction given in Table 26-2.

1. Calculate the water depth:

$$d_w = d_{total} - d_{ice} = 3.25\text{ m} - 0.38\text{ m} = 2.87\text{ m}$$

2. Calculate the time to cross the ice:

$$d = vt = \frac{c}{n}t$$

$$t_{ice} = \frac{n_{ice}\, d_{ice}}{c} = \frac{1.31(0.38\text{ m})}{3.00\times10^8\text{m/s}} = 1.7\text{ ns}$$

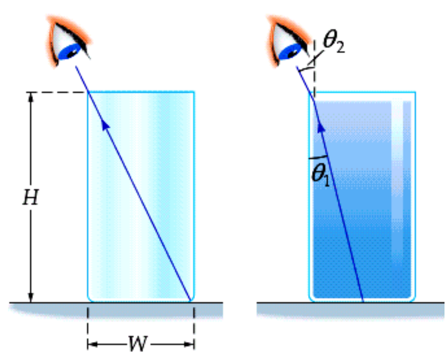
3. Calculate the time to cross the water:

$$t_w = \frac{n_w\, d_w}{c} = \frac{1.33(2.87\text{ m})}{3.00\times10^8\text{m/s}} = 12.7\text{ ns}$$

4. Add the two times together:

$$t_{total} = t_{ice} + t_w = 1.7\text{ ns} + 12.7\text{ ns} = \boxed{14.4\text{ ns}}$$

60. The image shows a person looking into an empty glass at an angle that allows her to barely see the bottom of the glass. When looking at the same angle after filling the glass with water, she can see the center of the bottom of the glass.



Use the empty glass to calculate the sine of the angle of refraction θ_2 in terms of the height H of the glass and the width of the bottom W. With the glass full, calculate sine of the angle of incidence θ_1 in terms of H and $W/2$. Then use Snell's Law (equation 26-11) to calculate the height of the glass.

1. Calculate $\sin\theta_2$:

$$\sin\theta_2 = \frac{W}{\sqrt{W^2 + H^2}}$$

2. Calculate $\sin\theta_1$:

$$\sin\theta_1 = \frac{W/2}{\sqrt{(W/2)^2 + H^2}} = \frac{W}{\sqrt{W^2 + 4H^2}}$$

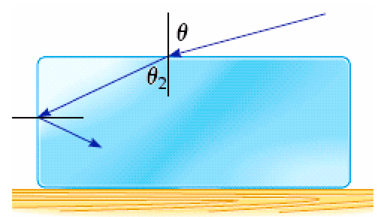
3. Solve Snell's Law for H:

$$n_{\text{air}} \sin \theta_{\text{i}} = n_{\text{w}} \sin \theta_{\text{refr}}$$

$$n_{\text{air}} \frac{W}{\sqrt{W^2 + H^2}} = n_{\text{w}} \frac{W}{\sqrt{W^2 + 4H^2}}$$

$$H = W\sqrt{\frac{{n_{\text{w}}}^2 - {n_{\text{air}}}^2}{4{n_{\text{air}}}^2 - {n_{\text{w}}}^2}} = 6.2 \text{ cm}\sqrt{\frac{(1.33)^2 - (1.00)^2}{4(1.00)^2 - (1.33)^2}} = \boxed{3.6 \text{ cm}}$$

64. The image shows a beam incident upon a horizontal glass surface. This beam refracts into the glass. When the refracted light hits the vertical surface it totally internally reflects.



From the figure, note that $\sin \theta_{\text{c}} = \sin(90° - \theta_2) = \cos \theta_2$. Use Snell's Law (equation 26-11) to write an equation relating the index of refraction and the refracted angle. Use equation 26-12 to write a second equation relating the refracted angle and index of refraction, where the sine of the critical angle is the cosine of the refracted angle. Square both of these equations and sum them to eliminate the angle and solve for the index of refraction.

1. (a) Write Snell's Law for the first refraction: $n \sin \theta_2 = \sin \theta$

2. Write equation 26-12 in terms of the refracted angle:

$$\sin \theta_{\text{c}} = \frac{n_{\text{air}}}{n} = \frac{1}{n} = \sin(90° - \theta_2) = \cos \theta_2$$

$$n \cos \theta_2 = 1$$

3. Sum the squares of the two equations and solve for *n:*

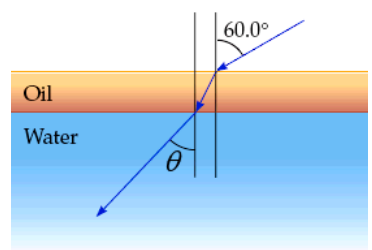
$$n^2 \sin^2 \theta_2 + n^2 \cos^2 \theta_2 = \sin^2 \theta + 1$$

$$n^2\left(\sin^2 \theta_2 + \cos^2 \theta_2\right) = n^2 = \sin^2 \theta + 1$$

$$n = \sqrt{\sin^2 \theta + 1} = \sqrt{\sin^2 70° + 1} = \boxed{1.4}$$

4. (b) Because the minimum index of refraction is related to the incident angle as $n = \sqrt{\sin^2 \theta + 1}$, decreasing θ will cause the minimum value of n to be $\boxed{\text{decreased}}$.

109. The figure shows a beam of light incident upon a film of oil at 60.0° from the vertical. The light is refracted in the oil and then refracted again as it enters the water.



Use Snell's Law (equation 26-11) to calculate the angle of refraction in the water. Because the air/oil and oil/water interfaces are parallel, the angle of refraction at the air/oil interface will equal the angle of incidence at the oil/water interface.

1. (a) Write Snell's Law at the air/oil interface: $n_{\text{air}} \sin \theta_1 = n_2 \sin \theta_2$

2. Write Snell's Law at the oil/water interface: $n_{\text{oil}} \sin \theta_2 = n_{\text{water}} \sin \theta$

3. Combine the two equations and solve for the angle of refraction in water:

$$n_{\text{air}} \sin \theta_1 = n_{\text{water}} \sin \theta$$

$$\theta = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{water}}} \sin \theta_1\right) = \sin^{-1}\left(\frac{1.00}{1.33} \sin 60.0°\right) = \boxed{40.6°}$$

4. (b) $\boxed{\text{The answer to part (a) does not depend upon the thickness of the oil film}}$, because θ depends only upon the original angle of incidence and the indices of refraction of air and water.