

Chapter 20:

Electric Potential and Electric Potential Energy

2. A $4.5 \mu\text{C}$ charge moves in a uniform electric field $\vec{E} = (4.1 \times 10^5 \text{ N/C}) \hat{x}$.

The change in electric potential energy of a charge that moves against an electric field is given by equation 20-1, $\Delta U = q_0 E d$. If the charge moves in the same direction as the field, the work done by the field is positive and $\Delta U = -W = -Fd = -q_0 E d$. If the charge moves in a direction perpendicular to the field, the work and change in potential energy are both zero. The change in potential energy from point A to point B is independent of the path the charge takes between those two points.

1. (a) The charge moves perpendicular to the field:

$$\Delta U = \boxed{0}$$

2. (b) The charge moves in the same direction as the field:

$$\Delta U = -q_0 E d = -(4.5 \times 10^{-6} \text{ C})(4.1 \times 10^5 \text{ N/C})(6.0 \text{ m}) = \boxed{-11 \text{ J}}$$

3. (c) The potential energy changes by an amount equal to the sum of paths 1 and 2 above:

$$\Delta U = 0 - 11 \text{ J} = \boxed{-11 \text{ J}}$$

9. The vertical electric field of the earth creates a potential difference between two different heights.

Because the electric field is uniform, the magnitude potential difference is simply the product of the field and the distance Δs in the direction of the field (equation 20-4).

Solve equation 20-4 for $|\Delta V|$:

$$|\Delta V| = E \Delta s = (100 \text{ V/m})(555 \text{ ft} \times 0.305 \text{ m/ft}) = \boxed{2 \times 10^4 \text{ V}} = 20 \text{ kV}$$

11. A potential difference across the gap of a spark plug produces a sufficiently large electric field to produce a spark.

The relationship between the magnitude of a uniform electric field and the potential difference along the direction of the field is given by equation 20-4. Solve this expression for the magnitude of the potential difference.

1. (a) Solve equation 20-4 for $|\Delta V|$:

$$|\Delta V| = E \Delta s = (3.0 \times 10^6 \text{ V/m})(0.025 \text{ in.} \times 0.0254 \text{ m/in.}) = \boxed{1.9 \text{ kV}}$$

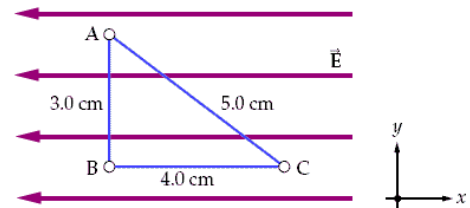
2. (b) The electric potential difference varies linearly with the separation Δs of the electrodes, so the required potential difference will increase if Δs is increased.

3. (c) Solve equation 20-4 for $|\Delta V|$:

$$|\Delta V| = E \Delta s = (3.0 \times 10^6 \text{ V/m})(0.050 \text{ in.} \times 0.0254 \text{ m/in.}) = \boxed{3.8 \text{ kV}}$$

12. A uniform electric field $\vec{E} = (-1200 \text{ N/C}) \hat{x}$ creates a change in the electric potential along the \hat{x} direction.

The change in electric potential is given by equation 20-4, $\Delta V = -E \Delta s$, where Δs is equal to Δx because the field points in the \hat{x} direction. There is no change in the electric potential along the \hat{y} and \hat{z} directions. The change in electric potential between any two points is independent of the path taken between those two points.



1. (a) $\Delta x = 0$ between points A and B:

$$\Delta V = V_B - V_A = -E \Delta x = \boxed{0}$$

2. (b) Solve equation 20-4 for ΔV between points B and C:

$$\begin{aligned} \Delta V &= V_B - V_C = -E \Delta x = -E(x_B - x_C) \\ &= -(-1200 \text{ N/C})(-0.040 \text{ m}) = \boxed{-48 \text{ V}} \end{aligned}$$

3. (c) Repeat for points C and A:

$$\begin{aligned} \Delta V &= V_C - V_A = -E \Delta x = -E(x_C - x_A) \\ &= -(-1200 \text{ N/C})(0.040 \text{ m}) = \boxed{48 \text{ V}} \end{aligned}$$

4. (d) No, we cannot determine the value of the electric potential at point A because the location of zero potential has not been established. We could always choose $V_A = 0$ if we wish!

19. An electron accelerates through a large potential difference in a TV picture tube.

The electron will accelerate toward a higher electric potential due to its negative charge. The change in potential energy is the charge times the potential difference (equation 20-2). The change in potential energy equals the gain in kinetic energy, which can then be used to find the speed.

1. Set the kinetic energy equal to the change in potential energy and solve for v :

$$\frac{1}{2}mv^2 = \Delta U = e\Delta V \Rightarrow v = \sqrt{\frac{2e\Delta V}{m}}$$

2. Find the speed of the electron:

$$v = \sqrt{\frac{2e\Delta V}{m}} = \sqrt{\frac{2(1.60 \times 10^{-19} \text{ C})(25,000 \text{ V})}{9.11 \times 10^{-31} \text{ kg}}} = \boxed{9.4 \times 10^7 \text{ m/s}}$$

22. The speed of a proton is changed as it moves through a potential difference.

Use conservation of energy, with the potential energy of the proton given by $U = qV = eV$, to determine the potential difference required to change the proton's speed and kinetic energy.

1. (a) Set $K_i + U_i = K_f + U_f$

$$\frac{1}{2}mv_i^2 + eV_i = \frac{1}{2}mv_f^2 + eV_f$$

and solve for ΔV :

$$\frac{1}{2}mv_i^2 - \frac{1}{2}mv_f^2 = e(V_f - V_i) = e\Delta V$$

$$\Delta V = \frac{m(v_i^2 - v_f^2)}{2e} = \frac{(1.673 \times 10^{-27} \text{ kg})(4.0 \times 10^5 \text{ m/s})^2 - 0}{2(1.60 \times 10^{-19} \text{ C})} = 840 \text{ V} = \boxed{0.84 \text{ kV}}$$

2. (b) Find ΔV for when $v_f = \frac{1}{2}v_i$:

$$\Delta V = \frac{m}{2e} \left[v_i^2 - \left(\frac{1}{2}v_i \right)^2 \right] = \frac{mv_i^2}{2e} \left(1 - \frac{1}{4} \right) = \frac{3mv_i^2}{8e}$$

$$= \frac{3(1.673 \times 10^{-27} \text{ kg})(4.0 \times 10^5 \text{ m/s})^2}{8(1.60 \times 10^{-19} \text{ C})} = 630 \text{ V} = \boxed{0.63 \text{ kV}}$$

3. (c) Set $K_i + U_i = \frac{1}{2}K_i + U_f$ and

$$K_i + eV_i = \frac{1}{2}K_i + eV_f$$

solve for ΔV :

$$\frac{1}{2}K_i = e\Delta V$$

$$\Delta V = \frac{K_i}{2e} = \frac{mv_i^2}{4e} = \frac{(1.673 \times 10^{-27} \text{ kg})(4.0 \times 10^5 \text{ m/s})^2}{4(1.60 \times 10^{-19} \text{ C})} = \boxed{0.42 \text{ kV}}$$

25. For the figure shown at right, it is given that $q_1 = +Q$ and that the electric potential at the center of the square is zero.

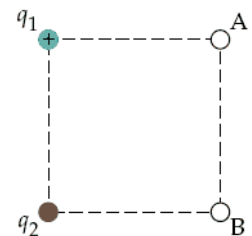
Use the expression for the potential due to a point charge (equation 20-7), together with the knowledge that the potentials due to different point charges simply add, to determine the value of q_2 and the potential at points A and B.

1. (a) The potential at the center of the square is the sum of the potentials due to q_1 and q_2 .

Each potential is given by $V = kq/r$ (equation 20-7). Because the two charges are equidistant from the center of the square, they must have equal and opposite charges in order for the potential there to be zero. We conclude that q_2 has the charge $-Q$.

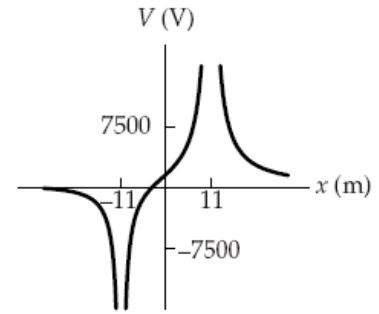
2. (b) The potential at point A is the sum of the potentials due to q_1 and q_2 . The charge q_1 has the same magnitude as q_2 but is closer to point A, and we conclude that the electric potential at point A will be positive.

3. (c) The potential at point B is the sum of the potentials due to q_1 and q_2 . The charge q_2 has the same magnitude as q_1 but is closer to point B, and we conclude that the electric potential at point B will be negative.



32. A plot of the electric potential as a function of position near the two opposite charges is shown at the right.

There is a point in the region $x < -11$ m where the potential vanishes because the positive charge has the greater magnitude, even though the negative charge is closer. Let q_1 be the $-2.2 \mu\text{C}$ charge at $x = -11$ m and q_2 be the $+4.1 \mu\text{C}$ charge at $x = +11$ m. Write the potential in this region as the sum of the potentials from the two charges, set it to zero, and solve for x .



1. (a) Write the potential as the sum of the potentials from the two charges in the region $x < -11$ m:

$$V = \frac{kq_1}{(-11\text{ m}) - x} + \frac{kq_2}{11\text{ m} - x} = 0$$

2. Rearrange and simplify the two terms:

$$\frac{kq_1}{11\text{ m} + x} = \frac{kq_2}{11\text{ m} - x}$$

3. Multiply both sides by the expressions in the denominators and rearrange:

$$q_1(11\text{ m} - x) = q_2(11\text{ m} + x)$$

$$(11\text{ m})(q_1 - q_2) = x(q_1 + q_2)$$

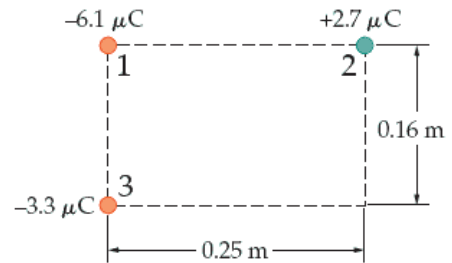
4. Solve the expression for x :

$$x = (11\text{ m}) \left(\frac{q_1 - q_2}{q_1 + q_2} \right) = (11\text{ m}) \left[\frac{(-2.2) - 4.1 \mu\text{C}}{(-2.2) + 4.1 \mu\text{C}} \right] = \boxed{-36\text{ m}}$$

5. (b) Because the electric field points “downhill” on the potential surface, we can see that the electric field is nonzero and positive at $x = -36$ m, the location where the potential is zero.

38. Three charges are arranged at the corners of a rectangle as indicated in the diagram at right.

The work required to move all three charges to infinity is equal to the change in the total electric potential energy of all three charges. This energy is the sum of terms like $U = kq_1q_2/r$ for each pair of charges in the system, as illustrated in active example 20-3. Sum the potential energies for each of the three pairs of charges in this system to find the work required to move all of them infinitely far from one another.



Let $r_{23} = \sqrt{(0.25\text{ m})^2 + (0.16\text{ m})^2} = 0.30\text{ m}$.

1. Find ΔU by summing the terms from each pair of charges:

$$W = \Delta U = 0 - \left(\frac{kq_1q_2}{r_{12}} + \frac{kq_1q_3}{r_{13}} + \frac{kq_2q_3}{r_{23}} \right) = -k \left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}} \right)$$

2. Factor out 10^{-12} C^2 from each product of charges and calculate W :

$$W = -(8990 \text{ N} \cdot \text{m}^2) \left[\frac{(-6.1)(2.7)}{0.25\text{ m}} + \frac{(-6.1)(-3.3)}{0.16\text{ m}} + \frac{(2.7)(-3.3)}{0.30\text{ m}} \right]$$

$$= \boxed{-0.27\text{ J}}$$

54. A capacitor is formed with two parallel metal plates separated by a dielectric layer.

Equation 20-15 gives the relationship between the capacitance of a parallel-plate capacitor, the plate area A , the plate separation d , and the dielectric constant κ . Solve the expression for the dielectric constant κ .

Solve equation 20-15 for κ and substitute $C = Q/V$:

$$\kappa = \frac{Cd}{\epsilon_0 A} = \frac{d}{\epsilon_0 A} \left(\frac{Q}{V} \right) = \frac{(0.25 \times 10^{-3} \text{ m})(1.2 \times 10^{-6} \text{ C})}{(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}) \pi (0.056 \text{ m})^2 (750 \text{ V})} = \boxed{4.6}$$

57. A capacitor is formed with two parallel metal plates separated by a dielectric layer.

Equation 20-15 gives the relationship between the capacitance of a parallel-plate capacitor, the plate area A , the plate separation d , and the dielectric constant κ . Solve this expression for the plate separation d .

1. (a) Solve equation 20-15 for d :

$$d = \frac{\kappa \epsilon_0 A}{C} = \frac{(1.00059) \left(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2} \right) (3.45 \times 10^{-4} \text{ m}^2)}{1630 \times 10^{-12} \text{ F}} = 1.87 \times 10^{-6} \text{ m} = \boxed{1.87 \mu\text{m}}$$

2. (b) Repeat with the new κ of paper:

$$d = \frac{\kappa \epsilon_0 A}{C} = \frac{(3.7) \left(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2} \right) (3.45 \times 10^{-4} \text{ m}^2)}{1330 \times 10^{-12} \text{ F}} = 6.9 \times 10^{-6} \text{ m} = \boxed{6.9 \mu\text{m}}$$

60. A capacitor is formed with two parallel metal plates separated by a small air gap.

Equation 20-15 gives the relationship between the capacitance of a parallel-plate capacitor, the plate area A , the plate separation d , and the dielectric constant κ . Solve this expression for the required plate area A . The maximum voltage that can be applied to the capacitor is $V_{\text{max}} = E_{\text{max}} d$, where E_{max} is the dielectric strength of air from Table 20-2.

1. (a) Solve equation 20-15 for A :

$$A = \frac{C d}{\kappa \epsilon_0} = \sqrt{\frac{(22 \times 10^{-12} \text{ F}) (2.6 \times 10^{-3} \text{ m})}{(1.00059) (8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2)}} = \boxed{0.0065 \text{ m}^2}$$

2. (b) Calculate $V_{\text{max}} = E_{\text{max}} d$:

$$V_{\text{max}} = E_{\text{max}} d = (3.0 \times 10^6 \text{ V/m}) (2.6 \times 10^{-3} \text{ m}) = \boxed{7.8 \text{ kV}}$$