Math and Math Education in Brazil

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Math in Brazil

Studying and experiencing pure and applied math and math education at Unicamp

Research and study in Brazil was an invaluable experience for a high school math teacher from Los Angeles, California. I was there to learn about their math education and curriculum in the secondary grades. I attended a few professional development sessions with other math teachers from various parts of the country. Also, in spending some time studying a few selected math topics, I developed a better sense for non-Euclidean geometry through independent study alongside several math graduate students from the United States and Brazil. I enhanced my own mathematical knowledge by learning about spherical and hyperbolic geometry through a few books (and textbooks) on the subject and its history, as well as from my advisor there, Dr. M. Helena Noronha. This report about my time in Brazil will be divided into three parts: secondary math education in Brazil, the professional development of prospective and in-service math teachers in Brazil, and my learning and sharing about non-Euclidean geometry.

Secondary math education in Brazil: The curriculum and its rationale, the textbooks, and its integrated approach.

The following report is a limited product of this educator's own research and experience lasting only four weeks, during which the secondary schools were on vacation. Three significant differences between secondary math education in Brazil and the US are that in Brazil math is integrated *and* required *every* year in high school and the middle school teachers are required a math degree. In the beginning, I spent a few days trying to find a list of National standards for each mathematical subject, thinking in terms of the common US structure (algebra, geometry, algebra 2…etc.). I searched the math department's library and the Internet. I learned that elementary school begins in first grade (1^{ra} serie) and lasts 5 years, during which basic math is taught (ensino basico). Middle school is known as "ensino fundamental" and lasts 4 years. The 3 years of High school is called "ensino medio" and its integrated math standards are organized according to year (1, 2, and 3) instead of specific mathematical areas.

After some research and with the help of a Brazilian math graduate student (whose mother is also a teacher), I finally found *Parametros Curriculares Nacionais* (PCN) *para a area de matematica no ensino fundamental* which expresses the rationale and objectives of the high school math curriculum as well as the teacher and student roles in the classroom –the Brazilian version of the National Council of Teachers of Mathematics' (NCTM) standards. The PCN for mathematics in high school states that they are guided by studies, research, practice, and debates done during the last few years but it was a 1979 version. The following descriptions were translated from Portuguese since there was not an English version of PCN. I had the help of a dictionary, several Brazilian students always standing by, and my fluency in Spanish (very similar to Brazilian Portuguese, if you did not know) most of all, allowed me to do this at a decent speed.

It is believed (ideally) that math should be developed according to its relevance and importance to the following areas: (1) the purpose of information (the address of information, to analyze everyday information, i.e. statistics, tables, and graphics). (2) Studying numbers and their operations (one should be able to do arithmetic and Algebra). (3) The study of big measurements (using interdisciplinary studies within arithmetic, Algebra, Geometry, and other areas). (4) The study of space and forms/figures in space (Geometry).

The following is a description of rationale as well as long-term objectives for the learner. Math is idealized as a rich source of knowledge from which problem-solvers and rational thinkers are taught. One should be able to identify knowledge that is related to *something*-as a medium for the problem solving of real-world situations. It is stated that math stimulates interest, curiosity, and the spirit of investigation to problem-solve. Math is used for quantitative and qualitative observations. Mathematical problem-solving skills and strategies should develop different forms of rational thinking and processes, like: intuition, induction, deduction, analogies, estimations, and approximations. It should use available technological resources in proper problem-solving situations. This all sounds nice but it is not carried out as intended because many teachers are not well trained. This is not uncommon

of US math teachers (including primary teachers).

The math PCN further outlines (and later rationalizes) how logic, history, citizenship, philosophy, social relevance, links to other disciplines, reality, scientific contributions, and its contribution to the intellectual development of the individual (my favorite) all play a role on the development and teaching of the subject. Other instructional tools like games, books, videos, calculators, computers, and manipulatives also play an important role in the learning and teaching process. They should be integrated in situations where the student can use these tools to better analyze, reflect, and formalize the mathematics. Math in citizenship is supposed to be relevant when it is necessary to calculate, measure, rationalize, argue, and treat information statistically. Another rationale is that the individual is trustworthy if he is capable of educated decision-making and has good problem-solving skills through math. Additionally "mathematical knowledge should be a result of human development and its constant relationship within a 'natural context', social and cultural. In that way, math will not be an irrelevant science but will transform itself into a discipline whose new information will solve scientific and technological problems."

Integrating logic permits an understanding of "process", possibility (existence), and the development (growth, process) of argument (debate, proof) and making generalizations. Additionally, it exercises the ability to justify and use formal proof. Integrating the history of numbers allows you to show how the system developed and grew. It allows you to show how the history of numbers is connected to the needs and worries of the people.

Brasil's ideal role of the math teacher is one of a mediator, facilitator, motivator, organizer, and evaluator and as such, they should aim to stimulate debate and dialogue over math topics in the classroom. They should support conversations about the processes and concepts related to math and provide students with information they would not have access to on their own. The teacher promotes cooperative learning so students work with and learn from each other. Teachers constantly evaluate to find out if the intended goals are being met. Last but not least, educators should provide new challenges

for themselves as well as the student.

I also had the opportunity to examine 2 different versions of high school math texts (2 versions labeled volumes 1-3, for a total of 6 texts) and analyzed the order of the lessons taught each year. In one particular **Volume 1** text (used in the 9th grade, first year of high school), the curriculum topics are expressed as follows:

1) Number systems
2) Progressions
3) Functions
4) Exponential Functions
5) Logarithms (and Log Functions)
6) Trigonometry

Volume 2, year 2, (10th grade):

1) Progressions (when the Volume 1 text does not cover it)
2) Matrices
3) Linear systems and determinants
4) Analytic combinatorics
5) Probability
6) Geometry

The Brazilian math curriculum objectives for the student and teacher are exemplary but not sufficient. It is difficult to find anything on differentiated learning. The curriculum does not address the needs of special education students or different intelligences. How do the teachers deal with students so poor that they attend school just for the free meal? In this case, even if more money was allocated to the schools, it still does not undo the poverty of the people around the schools. Finally, although the PCN claims to base itself on research, it is not enough since scientific research in the country barely existed until the 1960s. Unfortunately, I was not able to visit any classrooms and observe any lessons due to winter vacation.

The professional development of prospective and in-service math teachers in Brazil

I was fortunate to attend the University of Campinas (Unicamp) in Campinas, Sao Paulo because the math department there is known to be the leader in professional development (PD) for math educators in the country. I attended a string of teacher PD sessions for middle and high school

teachers. The sessions did not provide teachers with lesson plans or teaching methodologies, but instead presented various *interesting* problems different from those considered as a part of the high school math national standards (in the PCN) of Brazil. For example, in one session, we studied the algebraic analysis of magic squares and the proof of the nonexistence of a 2x2 one as well as the infinite number of solutions for a 3x3 square. In another session, the PD leader talked about The Spiral of Stanislav Ulam, Innumerability, prime numbers, and the concept of Infinity. All of these topics eventually lead to rigorous math computations and theory but none could be found in the textbooks I studied. In a different session, math games and puzzles were used as a way to draw the students' attention: Using curiosity as the way to lure their mathematical interest. *I* was personally enticed by the interesting topics of number theory and other areas, but other teachers expressed their concerns about the relevance of these workshops to a classroom. One voiced her difficulty with classroom-management, while another did not see the connection between Ulam's spiral and her classroom.

In another session, I spoke to a middle school teacher who majored in math and was now pursuing a master's degree in math education. She had recently attended a problem-solving PD session for middle school teachers and described her surprise when the other teachers could not solve a few 7^{th} and 8^{th} grade math problems posed earlier. "I had to explain it to them" she said. She felt that one of the problems in math education is the lack of math content knowledge of the teachers. This was particularly interesting because they are required a math degree.

Although the sessions were interesting, they did not focus on the major problems facing educators in Brasil, such as classroom management and a meager salary. Teachers in private schools have fewer problems with student behavior but still struggle with instruction and learning. Those in the public schools (and poorer areas) have more to deal with. Public school teachers could have used more instruction on the development of math concepts in each grade or how to reach the mind of a child that is forced to work for his family since elementary school. The educators expressed their desire to learn how to teach students of different social backgrounds and abilities--how to differentiate instruction.

Researching and learning about non-Euclidean geometry

The most fun of my research! The setting was ideal and inspiring. I studied alongside eight math graduate students from the US and eight from various universities in Brasil. The math department at Unicamp was generous to provide the group with our own classroom equipped with wireless Internet service and a rich math library downstairs. During our stay, each group of graduate students researched a specific problem and had weekly meetings with their advisors (professors from Unicamp). It was highly inspiring to see these students devoting their time to math and hear them report about their progress every Friday. I thought, if only my students at Birmingham High School in Van Nuys, California (L.A.U.S.D.) could see what it is like to *do* math-5 days per week for 4 weeks (and even that was not enough time!).

I have a B.S. in applied math and have taught high school geometry for 2 years but before this trip, knew very little of non-Euclidean geometry. The majority of my independent study was guided by *Experiencing Geometry: Euclidean and Non-Euclidean with History* (Henderson and Taimina) and *Euclidean and Non-Euclidean Geometries* (Noronha). I read about the history and leading mathematicians of non-Euclidean geometry (Gauss, Bolyai, Lobachevsky). The work done for this project has been so exciting and fulfilling that I wanted to design new lessons for my students and bring the same knowledge to them. I believe non-Euclidean geometry can be integrated within the regular curriculum. It would be a rich and realistic way of presenting analogous but contrasting material to Euclidean geometry.

It was professionally enriching to learn about the math language non-Euclidean geometry uses, for example linear algebra and matrix transformations. It was a new discovery to see so much linear algebra and vector calculus involved in the description of this geometry. I welcomed the need to look back and review these areas. The experience of finally understanding some important proofs in spherical and hyperbolic geometry is very exciting.

Hopefully, this year I will incorporate my knowledge of non-Euclidean geometry in my classes.

I plan to use it throughout the school year as a form of comparison and analogy to Euclidean geometry as well as a presentation of new ideas and the existence of other surfaces. My goal is to design a unit lesson on the introduction of non-Euclidean geometry for students and teachers. Part of it uses what I learned during my research in Brazil and another comes from a mini-course on the subject led by D. Henderson and D. Taimina at the M.A.A.-A.M.S. Joint Math conference earlier this year. Additionally, I plan to submit several abstracts to math conferences and share the multiple learning experiences in Brazil with others in the math community.

Conclusion

My experience in Brasil was wonderful and inspiring. I would love to continue a more focused research of math and its education in Brazil (as well as other countries). It is a valuable opportunity for the professional development of math teachers from many parts of the world. Math educators need time and money to develop their math teaching skills and content knowledge. They also need to be educated about what it means to do research, so I hope this type of program may be available to other teachers. Since math is the universal language, it seems logical it could create a strong link between educational institutions around the world. Teachers in the US could use the opportunity to travel abroad and experience the differences and similarities in math education, while their students need math teachers who inspire them to become future scientists and mathematicians.

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