

List of topics for the Topology Comprehensive Exam

1. Metric Spaces

- a) Open and closed sets in metric spaces
- b) Interior, boundary and closure
- c) Continuous functions
- d) Equivalence of metric spaces
- e) The metric topology
- f) Sequences in metric spaces
- g) Complete metric spaces
- h) Sequential compactness
- i) Limit point compactness

2. Topological Spaces

- a) Definition of a topological space
- b) Open and closed sets
- c) Neighborhoods
- d) Basis and subbasis of a topology
- e) Interior, closure and boundary of a set
- f) Continuity and topological equivalence, i.e. homeomorphism
- g) Topological invariants or topological properties
- h) Subspaces; Relative topology
- i) Comparison of topologies

3. Connectedness

- a) Connected spaces and their properties
- b) Connected subsets of the real line
- c) Connected components
- d) Connectedness of the continuous image of a connected space
- e) Application of connectedness; for example The Intermediate Value Theorem
- f) Path connected spaces
- g) Path connected vs. connected

4. Compactness

- a) Definition of compactness in Euclidean, metric and general topological spaces.
- b) Compactness of closed subsets of a compact space
- c) Compact subsets of a Hausdorff space are closed
- d) Continuous images of a compact space
- e) One-point compactification
- f) Locally compact spaces

5. Separation Properties

- a) Hausdorff, regular and normal spaces
- b) Separation by continuous functions

6. Product Spaces

- a) Finite products'
- b) Arbitrary products with the product topology
- c) The product topology vs. the box topology

7. Fundamental Groups

- a) Homotopic maps
- b) Definition of the fundamental group of a space
- c) Change-of-basepoint isomorphism
- d) The fundamental group of the circle
- e) Van Kampen's Theorem
- f) Retractions and deformation retractions

References

- Topology, a first course by J. Munkres
- Basic Topology, by M. A. Armstrong
- Topology, by K. Janich
- Foundations of Topology by C. Wayne Patty
- Introduction to Metric and Topological Spaces by W. Sutherland
- Algebraic Topology, by Allen Hatcher (Chapters 0 and 1)
- Principles of Topology by Fred H. Croom
- General Topology by Stephen Willard

The books are available in the Oviatt Library.