Real Analysis
Topics for the Masters Degree Qualifying Exam

The list below contains core topics and optional topics. Thorough mastery of the Core Topics is sufficient to pass the comprehensive real analysis exam. However, some problems associated with the list of Optional Topics may also appear, and therefore knowledge of some or all of that material gives the test taker more choices of problems to attempt.

CORE TOPICS

1. MEASURES AND σ–ALGEBRAS
1.1 Borel sets, Lebesgue σ–algebra on the real line, existence of non measurable sets, complete σ–algebras, σ–algebras generated by functions
1.2 General properties of measures, counting measure, Lebesgue measure on the real line, cumulative distribution functions, outer measure, Stieltjes measures on the real line, finite measures and σ–finite measures

2. MEASURABLE FUNCTIONS
2.1 Step functions, simple functions, continuous functions, measurable functions, functions of bounded variation, the Cantor Function
2.2 Convex Functions and Jensen's inequality
2.3 Egeroff's Theorem, Lusin's Theorem

3. INTEGRATION
3.1 Lebesgue integral
3.2 Monotone convergence theorem, Fatou's Lemma, Lebesgue Dominated Convergence Theorem
3.3 Absolutely continuous functions and the Fundamental Theorem of Calculus
3.4 Necessary and sufficient conditions for Riemann integrability

4. L^p SPACES
4.1 Construction of L^p spaces and completeness
4.2 Minkowski and Holder inequalities
4.3 Riesz Representation Theorem for bounded linear functionals

OPTIONAL TOPICS

5. PRODUCT MEASURES
5.1 Finite product σ–algebras, finite products of measures, Lebesgue measure on R^n.
5.2 Fubini and Tonelli theorems

6. SIGNED MEASURES
6.1 Hahn Decomposition Theorem
6.2 Jordan Decomposition Theorem