MATH 460, 462, and 560 : ABSTRACT ALGEBRA
Topics for Qualifying Exams and
Some Suggested Books

Groups
1. Definitions, Examples, Subgroups, Cosets. Normal subgroups, Examples, Group homomorphisms and isomorphisms, Kernels, Homomorphism theorems for groups.
2. Symmetric group, parity of a permutation, factorization of a permutation into disjoint cycles, conjugates of a permutation.
3. Conjugacy classes, class equation, applications to Z(G) for G a p-group and to groups of order p^2.
4. Group Actions on sets, counting orbit sizes.
5. Sylow Theorems.
6. Simple groups, Solvable groups.
7. Fundamental theorem for finitely generated abelian groups.

Rings
1. Definitions, Examples, Subrings, ideals, ring homomorphisms and isomorphisms, Kernels, Homomorphism theorems for rings.
2. Rings of fractions
3. Chinese Remainder Theorem
4. Euclidean Domains, Principal Ideal Domains, Unique Factorization Domains.
5. Polynomial rings, irreducibility theorems
6. Prime ideals and maximal ideals, Integral domains, Rings of quotients of integral domains.
7. Noetherian rings.

Fields
1. Roots of polynomials over extension fields of F, existence of roots, uniqueness up to F-isomorphism of F(v), where v is a root of a given irreducible polynomial. Splitting field of a polynomial over F, uniqueness up to F-isomorphism.
2. Cyclotomic polynomials, factoring cyclotomic polynomials over Z, roots of unity.
3. Finite fields: existence and uniqueness of fields of cardinality p^n for any n.
4. Normal and separable extensions, Fundamental theorem of Galois theory.
5. Galois groups of cyclotomic extensions, kummer extensions.
6. Solvability by radicals.

Modules
1. Basic definitions and examples
2. Quotient Modules and Module Homomorphisms
3. Generations of Modules, Direct sums and Free Modules.
4. Noetherian modules.
5. Structure theory of finitely generated modules over Z---finite abelian groups.

Vector Spaces and Linear Transformations.

1. Finite-dimensional vector spaces, subspaces, linear independence, bases, dimension, direct sums, quotients.
2. Linear transformations, kernel, image, bases and dimensions of kernel and image, invertible transformations, representation of linear transformations by matrices, change of coordinate matrices.
3. Minimal polynomial, characteristic polynomial, Cayley-Hamilton theorem, rational canonical forms, Jordan canonical forms, eigenvalues and eigenvectors, algebraic vs. geometric multiplicity, diagonalizability, invariant subspaces, cyclic subspaces.
4. Inner products on real and complex vector spaces, Cauchy-Schwarz and triangle inequalities, norms of vectors and matrices, orthogonal bases, Gram-Schmidt process, rotation matrices, normal matrices including hermitian, skew-hermitian and unitary matrices, diagonalization of normal matrices.

Some Suggested Books

1. Abstract Algebra by Dummit and Foote
2. Algebra by Hungerford
3. Topics in Algebra by Herstein
4. Linear Algebra Done Right by Axler
5. Linear Algebra by Friedberg, Insel, and Spence