

TOPICS FOR THE COMPREHENSIVE EXAM
Calculus on Manifolds

1 Integration in \mathbb{R}^n

1. Measure and Content Zero.
2. Integrable Functions.
3. Fubini's Theorem and Liebnitz's rule.
4. Partitions of Unity and Change of Variables.

2 Differential Forms in \mathbb{R}^n

1. Multilinear Algebra.
2. Basic Concepts: Exterior Form, Exterior Product, Exterior Differential.
3. The Hodge star operator.
4. Gradient, Divergence, Laplacian, Curl.

3 Differentiable Manifolds

1. Differentiable manifolds.
2. The tangent bundle.
3. Immersions and embeddings.
4. Orientable manifolds.
5. Manifolds with boundary.

4 Integration on Manifolds

1. Stokes' Theorem on Manifolds.
2. The Green and the Divergence (Gauss) Theorems.
3. The Stokes' Theorem for surfaces of \mathbb{R}^3 with boundary.
4. Poincaré's lemma.
5. Potential Theory in \mathbb{R}^3 : Harmonic functions, Green's identities, and the Maximum Principle.

5 Recommended Textbooks

1. do Carmo, M. P. *Differential Forms and Applications*, Chapters 1, 3, and 4. Springer-Verlag, 1994.
2. Spivak, M. *Calculus on Manifolds*, Chapters 3 and 4. Addison-Wesley Publishing Company.
3. Wade, W.R., *An Introduction to Analysis*, Chapters 12 and 13. Prentice Hall, UpperSaddle River, New Jersey, 2004.
4. Munkres, J., *Analysis on Manifolds*, Chapters 3, 4 ,5, 6, and 7. Westview Press, Perseus Book Group.